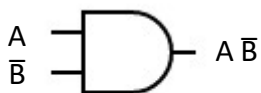


Simplifying Logic Circuit

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Fundamental Products

- Two input Signals
- ANDing two variables and their compliments



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Fundamental Products

A	B	$\bar{A}\bar{B}$	$\bar{A}B$	$A\bar{B}$	AB
0	0	1	0	0	0
0	1	0	1	0	0
1	0	0	0	1	0
1	1	0	0	0	1

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Fundamental Products of Two Variables

A	B	Fundamental product
0	0	$\bar{A}\bar{B}$
0	1	$\bar{A}B$
1	0	$A\bar{B}$
1	1	AB

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Fundamental Products of Three Variables

A	B	C	Fundamental Product
0	0	0	$\bar{A} \bar{B} \bar{C}$
0	0	1	$\bar{A} \bar{B} C$
0	1	0	$\bar{A} B \bar{C}$
0	1	1	$\bar{A} B C$
1	0	0	$A \bar{B} \bar{C}$
1	0	1	$A \bar{B} C$
1	1	0	$A B \bar{C}$
1	1	1	$A B C$

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Sum of Product

A	B	Y	FP
0	0	0	
0	1	0	
1	0	1	$A \bar{B}$
1	1	1	AB

$$Y = A\bar{B} + AB$$

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Sum of Product

A	B	Y	FP
0	0	0	
0	1	1	$\bar{A}B$
1	0	1	$A\bar{B}$
1	1	0	

$$Y = \bar{A}B + A\bar{B}$$

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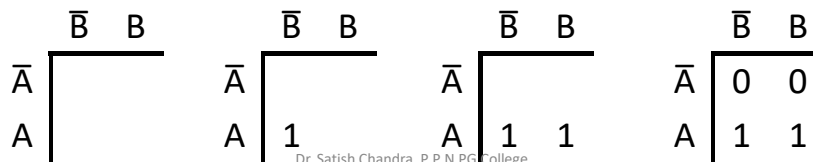
Sum of Product (SOP)

- Find the fundamental product corresponding to each 1 output in the Truth Table.
- OR the fundamental product.

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Karnaugh Map

A	B	Y	FP
0	0	0	
0	1	0	
1	0	1	$A\bar{B}$
1	1	1	AB



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Karnaugh Map

- Karnaugh Map differ from a truth table.
- A truth table shows the output for each input condition.
- A Karnaugh map shows the fundamental products needed to produce the output 1s for corresponding input conditions.

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Karnaugh Map

A	B	Y	FP
0	0	0	
0	1	1	$\bar{A}B$
1	0	1	$A\bar{B}$
1	1	0	

	\bar{B}	B
\bar{A}		
A		

	\bar{B}	B
\bar{A}	1	
A	1	

	\bar{B}	B
\bar{A}	1	
A	1	

	\bar{B}	B
\bar{A}	0 1	
A	1 0	

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Karnaugh Map – Three Variable

A	B	C	Y	FP
0	0	0	0	
0	0	1	0	
0	1	0	1	$\bar{A}B\bar{C}$
0	1	1	0	
1	0	0	0	
1	0	1	0	
1	1	0	1	$AB\bar{C}$
1	1	1	1	ABC

	\bar{C}	C
$\bar{A}\bar{B}$		
$\bar{A}B$		
AB		
$A\bar{B}$		

	\bar{C}	C
$\bar{A}\bar{B}$		
$\bar{A}B$	1	
AB	1 1	
$A\bar{B}$		

	\bar{C}	C
$\bar{A}\bar{B}$	0 0	
$\bar{A}B$	1 0	
AB	1 1	
$A\bar{B}$	0 0	

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Karnaugh Map – Four Variable

$\bar{A}\bar{B}$	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}B$				
AB				
$A\bar{B}$				

The vertical column and horizontal row are labelled in Gray Code progression in Karnaugh Map

A	B	C	D	Y	FP
0	0	0	0	0	
0	0	0	1	1	$\bar{A}\bar{B}C\bar{D}$
0	0	1	0	0	
0	0	1	1	0	
0	1	0	0	0	
0	1	0	1	0	
0	1	1	0	1	$\bar{A}BC\bar{D}$
0	1	1	1	1	$\bar{A}BCD$
1	0	0	0	0	
1	0	0	1	0	
1	0	1	0	0	
1	0	1	1	0	
1	1	0	0	0	
1	1	0	1	0	
1	1	1	0	1	$ABC\bar{D}$
1	1	1	1	0	

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Karnaugh Map – Four Variable

$\bar{A}\bar{B}$	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}B$				
AB				
$A\bar{B}$				

The vertical column and horizontal row are labelled in Gray Code progression in Karnaugh Map

A	B	C	D	Y	FP
0	0	0	0	0	
0	0	0	1	1	$\bar{A}\bar{B}C\bar{D}$
0	0	1	0	0	
0	0	1	1	0	
0	1	0	0	0	
0	1	0	1	0	
0	1	1	0	1	$\bar{A}BC\bar{D}$
0	1	1	1	1	$\bar{A}BCD$
1	0	0	0	0	
1	0	0	1	0	
1	0	1	0	0	
1	0	1	1	0	
1	1	0	0	0	
1	1	0	1	0	
1	1	1	0	1	$ABC\bar{D}$
1	1	1	1	0	

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Karnaugh Map – Four Variable

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	1	0	0
$\bar{A}B$	0	0	1	1
AB	0	0	0	1
$A\bar{B}$	0	0	0	0

A	B	C	D	Y	FP
0	0	0	0	0	
0	0	0	1	1	$\bar{A}\bar{B}\bar{C}D$
0	0	1	0	0	
0	0	1	1	0	
0	1	0	0	0	
0	1	0	1	0	
0	1	1	0	1	$\bar{A}BC\bar{D}$
0	1	1	1	1	$\bar{A}BCD$
1	0	0	0	0	
1	0	0	1	0	
1	0	1	0	0	
1	0	1	1	0	
1	1	0	0	0	
1	1	0	1	0	
1	1	1	0	1	$ABC\bar{D}$
1	1	1	1	0	

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Karnaugh Map Simplification – Pairs

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	0	0	0
$\bar{A}B$	0	0	0	0
AB	0	0	1	1
$A\bar{B}$	0	0	0	0

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	0	0	0
$\bar{A}B$	0	0	0	0
AB	0	0	1	1
$A\bar{B}$	0	0	0	0

Eliminate the variable that goes from complemented to uncomplemented form (or vice versa).

$$y = ABC$$

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Karnaugh Map Simplification – Pairs

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	0	0	0
$\bar{A}B$	0	0	0	0
AB	0	0	0	1
$A\bar{B}$	0	0	0	1

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	0	1	0
$\bar{A}B$	0	0	1	0
AB	0	0	0	0
$A\bar{B}$	0	0	0	0

$$y = AC\bar{D}$$

$$y = \bar{A}CD$$

Eliminate the variable that changes form

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Karnaugh Map Simplification – Pairs

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	0	0	0
$\bar{A}B$	0	0	0	0
AB	0	0	0	0
$A\bar{B}$	1	1	0	0

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	0	0	0
$\bar{A}B$	0	1	1	0
AB	1	0	0	0
$A\bar{B}$	1	0	0	0

$$y = A\bar{B}\bar{C}$$

$$y = A\bar{C}\bar{D} + \bar{A}BD$$

One variable or their complement drop out of the Boolean equation.

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Karnaugh Map Simplification – Quads

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	0	0	0
$\bar{A}B$	0	0	0	0
AB	1	1	1	1
$A\bar{B}$	0	0	0	0

$$y = AB$$

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	0	0	0
$\bar{A}B$	0	0	0	0
AB	0	0	1	1
$A\bar{B}$	0	0	1	1

$$y = AC$$

Two variable and their complements drop out of the Boolean equation.

Karnaugh Map Simplification – Octet

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	0	0	0
$\bar{A}B$	0	0	0	0
AB	1	1	1	1
$A\bar{B}$	1	1	1	1

$$y = A$$

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	0	0	0
$\bar{A}B$	0	0	0	0
AB	1	1	1	1
$A\bar{B}$	1	1	1	1

$$y = A\bar{C} + AC = A$$

Three variable and their complements drop out of the Boolean equation.

Karnaugh Map Simplification – Nonoverlapping Groups

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	1	1	1
$\bar{A}B$	0	0	0	1
AB	1	1	0	1
$A\bar{B}$	1	1	0	1

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	1	1	1
$\bar{A}B$	0	0	0	1
AB	1	1	0	1
$A\bar{B}$	1	1	0	1

$$y = \bar{A}\bar{B}D + A\bar{C} + C\bar{D}$$

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Encircle the octets first, the quads second, and the pairs last.

Karnaugh Map Simplification – Overlapping Groups

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	0	0	0
$\bar{A}B$	0	1	0	0
AB	1	1	1	1
$A\bar{B}$	1	1	1	1

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	0	0	0
$\bar{A}B$	0	1	0	0
AB	1	1	1	1
$A\bar{B}$	1	1	1	1

$$y = A + B\bar{C}D$$

$$y = A + \bar{A}B\bar{C}D$$

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Karnaugh Map Simplification – Rolling

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	0	0	0
$\bar{A}B$	1	0	0	1
AB	1	0	0	1
$A\bar{B}$	0	0	0	0

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	0	0	0
$\bar{A}B$	1	0	0	1
AB	1	0	0	1
$A\bar{B}$	0	0	0	0

$$y = \bar{B}\bar{C}\bar{D} + B\bar{C}\bar{D}$$

$$y = B\bar{D}$$

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Karnaugh Map Simplification – Rolling & Overlapping

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1	1	0	0
$\bar{A}B$	1	1	0	1
AB	1	1	0	1
$A\bar{B}$	1	1	0	0

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1	1	0	0
$\bar{A}B$	1	1	0	1
AB	1	1	0	1
$A\bar{B}$	1	1	0	0

$$y = \bar{C} + B\bar{C}\bar{D}$$

$$y = \bar{C} + B\bar{D}$$

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Karnaugh Map Simplification – Rolling & Overlapping

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1	1	0	1
$\bar{A}B$	1	1	0	1
AB	1	1	0	0
$A\bar{B}$	1	1	0	1

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1	1	0	1
$\bar{A}B$	1	1	0	1
AB	1	1	0	0
$A\bar{B}$	1	1	0	1

$$y = \bar{C} + \bar{A}\bar{C}\bar{D} + ABC\bar{D}$$

$$y = \bar{C} + \bar{A}\bar{D} + A\bar{B}\bar{D}$$

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Encircle the octets first, the quads second, and the pairs last.

Karnaugh Map Simplification – Rolling & Overlapping

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1	1	0	1
$\bar{A}B$	1	1	0	1
AB	1	1	0	0
$A\bar{B}$	1	1	0	1

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1	1	0	1
$\bar{A}B$	1	1	0	1
AB	1	1	0	0
$A\bar{B}$	1	1	0	1

$$y = \bar{C} + \bar{A}\bar{C}\bar{D} + ABC\bar{D}$$

$$y = \bar{C} + \bar{A}\bar{D} + \bar{B}C\bar{D}$$

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Karnaugh Map Simplification – Eliminating Redundant Group

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	0	1	0
$\bar{A}B$	1	1	1	0
AB	0	1	1	1
$A\bar{B}$	0	1	0	0

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	0	1	0
$\bar{A}B$	1	1	1	0
AB	0	1	1	1
$A\bar{B}$	0	1	0	0

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Karnaugh Map Simplification – Eliminating Redundant Group

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	0	1	0
$\bar{A}B$	1	1	1	0
AB	0	1	1	1
$A\bar{B}$	0	1	0	0

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	0	1	0
$\bar{A}B$	1	1	1	0
AB	0	1	1	1
$A\bar{B}$	0	1	0	0

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Encircle the octets first, the quads second, and the pairs last.

Karnaugh Map - Summary

- Enter a 1 on the the K-map for each FP that produces a 1 output in the TT. Enter 0s elsewhere.
- Encircle the octets, quads and pairs. Remember to roll and overlap to get the largest group possible.
- If any isolated 1s remain, encircle each.
- Review the groups, and eliminate any redundant group.
- Write the Boolean equation by ORing the products corresponding to the encircled groups.

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Product of Sum

- The sum of products equation is

$$y = \overline{A}\overline{B} + AB + AC$$

	$\overline{C}\overline{D}$	$\overline{C}D$	CD	$C\overline{D}$
$\overline{A}\overline{B}$	1	1	1	1
$\overline{A}B$	0	0	0	0
AB	1	1	1	1
$A\overline{B}$	0	0	1	1

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Product of Sum

- Complement 0s and 1s to get new map

$$\bar{y} = \bar{A}\bar{B} + A\bar{B}\bar{C}$$

- Take the complement

$$y = \overline{\bar{A}\bar{B} + A\bar{B}\bar{C}}$$

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	0	0	0
$\bar{A}B$	1	1	1	1
AB	0	0	0	0
$A\bar{B}$	1	1	0	0

- Applying De Morgan's Theorem

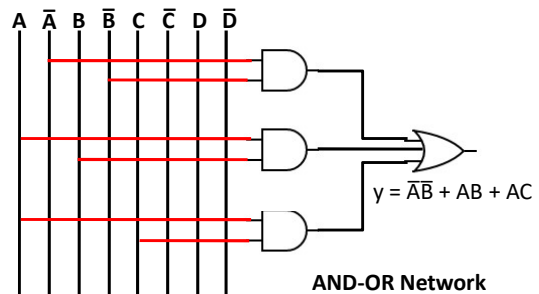
$$y = (A + \bar{B})(\bar{A} + B + C)$$

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Product of Sum

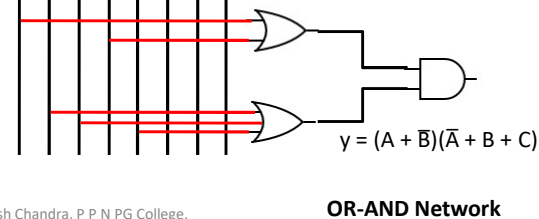
Sum of Product (SoP)

$$y = \bar{A}\bar{B} + AB + AC$$



Product of Sum (PoS)

$$y = (A + \bar{B})(\bar{A} + B + C)$$



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Product of Sum

- Given the TT for y , draw the K-map for FP.
- Simplify to get a SoP equation for \bar{y} .
- Apply the De Morgan's theorems to get a PoS equation.
- Draw the logic circuit, which consists of OR gates driving an AND gate.

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