Arithmetic fundamentals of Number systems

Satish Chandra
Associate Professor

Outline

- Different number systems
- Why use different ones?
- Binary / Octal / Hexadecimal
- Conversions
- Binary arithmetic

Number Systems

Four number systems:

- Decimal (10)
- Binary (2)
- Octal (8)
- Hexadecimal (16)

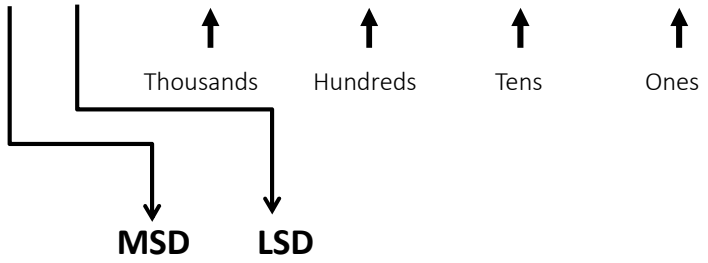
Decimal Number System

- It has a base (or Radix) of ten.
- It uses ten different symbols to represent numbers.
- These are; 0, 1, 2, 3, 4, 5, 6, 7, 8, & 9.
- Numbers above ten are combination of its ten unique symbol.

Decimal Numbers

Position Value

$1439 = 1 \times 10^3 + 4 \times 10^2 + 3 \times 10^1 + 9 \times 10^0$



- Base or Radix = 10

Binary Number System

- Computers work only on two states
  - On
  - Off
- Basic memory elements hold only two states
  - Zero / One
- Thus a number system with two elements {0,1}
- Base or Radix = 2
- A binary digit – bit !
- Eight bit – One byte !!

Binary Number Progression

Decimal	Binary
0	0
1	1
2	10
3	11
4	100
5	101
6	110
7	111
8	1000
9	1001
10	1010
11	1011
12	1100
13	1101
14	1110
15	1111
16	10000

Binary → Decimal

$1101 = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$

$= 1 \times 8 + 1 \times 4 + 0 \times 2 + 1 \times 1$

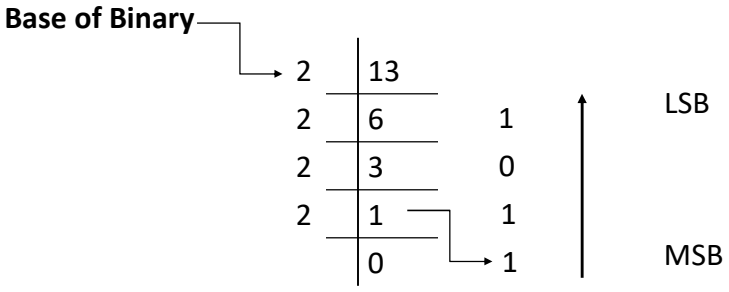
$= 8 + 4 + 0 + 1$

MSB **LSB**

$(1101)_2 = (13)_{10}$

1, 2, 4, 8, 16, 32, 64, 128, 256, 512, …

Decimal → Binary



$(13)_{10} = (1101)_2$

Octal Number System

- It has a base (or Radix) of eight.
- It uses eight different symbols to represent numbers.
- These are; 0, 1, 2, 3, 4, 5, 6 & 7.
- Numbers above eight are combination of its eight unique symbol.

Binary Number Progression

Decimal	Binary	Octal
0	0	**0**
1	1	**1**
2	10	**2**
3	11	**3**
4	100	**4**
5	101	**5**
6	110	**6**
7	111	**7**
8	1000	**10**
9	1001	**11**
10	1010	**12**
11	1011	**13**
12	1100	**14**
13	1101	**15**
14	1110	**16**
15	1111	**17**
16	10000	**20**

Octal → Decimal

$137 = 1 \times 8^2 + 3 \times 8^1 + 7 \times 8^0$

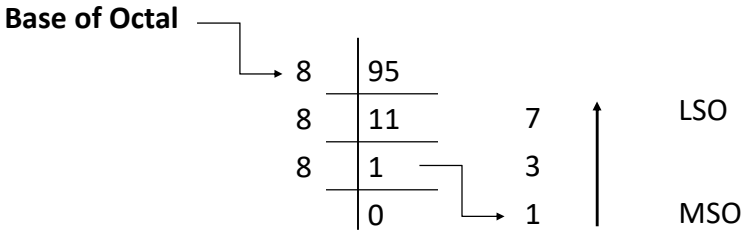
$= 1 \times 64 + 3 \times 8 + 7 \times 1$

$= 64 + 24 + 7$

$(137)_8 = (95)_{10}$

- Digits used in Octal number system – 0 to 7

Decimal → Octal



$(95)_{10} = (137)_8$

Hexadecimal Number System

- It has a base (or Radix) of sixteen.
- It uses sixteen different symbols (alphanumeric) to represent numbers.
- These are; 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E & F.
- Numbers above sixteen are combination of its sixteen unique symbol.

Binary Number Progression

Decimal	Binary	Octal	Hexadecimal
0	0	0	**0**
1	1	1	**1**
2	10	2	**2**
3	11	3	**3**
4	100	4	**4**
5	101	5	**5**
6	110	6	**6**
7	111	7	**7**
8	1000	10	**8**
9	1001	11	**9**
10	1010	12	**A**
11	1011	13	**B**
12	1100	14	**C**
13	1101	15	**D**
14	1110	16	**E**
15	1111	17	**F**
16	10000	20	**10**

Hex → Decimal

BAD = B x 16^2 + A x 16^1 + D x 16^0

= 11 x 16^2 + 10 x 16^1 + 13 x 16^0

= 11 x 256 + 10 x 16 + 13 x 1

= 2816 + 160 + 13

$(BAD)_{16} = (2989)_{10}$

A = 10, B = 11, C = 12, D = 13, E = 14, F = 15

Decimal → Hex

Base of Hexadecimal

16	2989			
16	186	13	B	LSH
16	11	10	A	
	0	11	D	MSH

$(2989)_{10} = (BAD)_{16}$

Why octal or hex?

- Ease of use and conversion
- Three bits make one octal digit

  111 010 110 101

  7 2 6 5 => 7265 in octal

- Four bits make one hexadecimal digit

  1110 1011 0101 → 4 bits = nibble

  E B 5 => EB5 in hex
- Useful in machine language (programming of microprocessor).

Binary Arithmetic

- Addition
- Subtraction

Addition

Like normal decimal addition

B

A

+	0	1
0	0	1
1	1	10

0101 (5)

+ 1001 (9)

1110 (14)

The carry out of the MSB is neglected

Subtraction

Like normal decimal subtraction

B

A

-	0	1
0	0	11
1	1	0

1001 (9)

- 0101 (5)

0100 (4)

A borrow (shown in red) from the MSB implies a negative