Nicol Prism as Polarizer and Analyzer: Nicol prism can be used both as polarizer and as an analyzer. When two Nicol prisms are mounted co-axially, then the first Nicol prism N_1 which produces plane polarized light is called the ***polarizer***, while the second Nicol prism N_2 which analyzes the incoming light is called the ***analyzer***.

When the principal sections of the two Nicols are parallel, then the vibrations in the *e-ray* which are in the principal section of the polarizer are also in the principal section of the analyzer (Figure 12). Therefore, the *e-ray* from N_1 is freely transmitted by N_2. This setting of Nicol prisms are known as ***parallel Nicols***. The intensity of the field of view is *maximum*.

When the analyzer is rotated about its axis through 90° from previous position, the principal sections of the Nicols are mutually perpendicular (Figure 13). The vibrations of *e-ray* from the polarizer, enters the analyzer as *o-ray* and therefore, totally internally reflected from the balsam. Thus, no light is transmitted by this setting of Nicol prism and the intensity of the field of view is zero. This setting is known as ***crossed Nicols***.

After a rotation of 180° the principal sections are again *parallel* and *e-ray* are again transmitted by the analyzer and so the intensity is maximum. After a rotation of 270° Nicols are again *crossed*, no light is transmitted and intensity is therefore, again zero.

3.5 RETARDATION PLATES

A plate cut from a doubly refracting crystal by sections parallel to the optic axis and used to introduce the desired phase difference between the *o-ray* and *e-ray*, when they transmit normally through it, is called retardation plates. These are two most useful retardation plates, namely (a) Half wave plate, and (b) Quarter wave plate.

Let the thickness of any retardation plate be t in the direction of propagation, n_o and n_e are the refractive indices for the *o-ray* and *e-ray*, respectively. Within the plate, the optical path for the *e-ray* is simply $n_o t$ and that for the *o-ray* is $n_e t$. The path difference is, therefore

$\Delta = (n_o - n_e)t$ as, $n_o > n_e$ For negative crystals (such as calcite)

$\Delta = (n_e - n_o)t$ as, $n_e > n_o$ For positive crystals (such as quartz)

(a) **Half Wave Plate:** It is of such a thickness that in passing through the plate a relative phase difference of π introduce in between the *o-ray* and *e-ray* and accordingly one ray drops behind the other by just one half of a wavelength (i.e. $\lambda/2$). This is why it is known as ***half wave plate***. This plate is being used in *Laurent half shade polarimeter*. The thickness of the half wave plate may be computed by placing , $\Delta = \lambda/2$

$t = \frac{\lambda}{2(n_o - n_e)}$ as, $n_o > n_e$ For negative crystals (such as calcite)

$t = \frac{\lambda}{2(n_e - n_o)}$ as, $n_e > n_o$ For positive crystals (such as quartz)

(b) **Quarter Wave Plate:** It is of such a thickness that in passing through the plate a relative phase difference of $\pi/2$ introduce in between the *o-ray* and *e-ray* and accordingly one ray drops behind the other by just one quarter of a wavelength (i.e. $\lambda/4$). This is why it is known as ***quarter wave plate***. This plate is being used for *producing and detecting circularly and elliptically polarized light*. The thickness of the quarter wave plate may be computed by placing , $\Delta = \lambda/4$

$t = \frac{\lambda}{4(n_o - n_e)}$ as, $n_o > n_e$ For negative crystals (such as calcite)

$t = \frac{\lambda}{4(n_e - n_o)}$ as, $n_e > n_o$ For positive crystals (such as quartz)

So, the quarter wave plate is only half as thick as the half wave plate.

3.6 BABINET COMPENSATOR

The use of any retardation plate is limited only to a narrow range of wavelengths, for which the path difference between e-ray and o-ray on transmission through it is fixed. The Babinet's compensator is an apparatus which has no such limitation of wavelength. It can introduce any desired path difference between the two components.

The Babinet compensator consists of two quartz wedge of equal small acute angles. The optic axis in one wedge is parallel to, while in the other perpendicular to the longer edge of their free rectangular faces. The optic axes are mutually perpendicular when the wedges are placed in contact with each other so as to form a thin plate of rectangular ABCD (Figure 14). The instrument has fixed cross-wires in front of the upper wedge and a micrometer screw to displace the lower wedge relative to the upper fixed wedge thus thickness can be varied.

Suppose, a parallel monochromatic plane polarized beam of light incident normally on the compensator. Just on entering the first wedge the incident vibration is resolved into two components, e-component parallel to the optic axis and o-component perpendicular to it (since quartz is a positive crystal). Since, the optic axis of the second wedge is perpendicular to the first; the *o-ray* and *e-ray* in the first wedge are transmitted as *e-ray* and *o-ray*, respectively.

Let us consider a point on the compensator where the upper wedge is of thickness h_1 and the lower wedge of thickness, h_2 (as shown in Figure 15). The optical path difference introduces between *e-ray* and *o-ray* in the first wedge is $(n_e - n_o)h_1$, and in second wedge is $(n_o - n_e)h_2$. Hence, the total optical path difference is, $\Delta = (n_e - n_o)(h_1 - h_2)$.

In this way, the $(h_1 - h_2)$ can be made to have any desired value by the micrometer screw. Hence, any path difference can be introduced with the compensator. That is why, Babinet compensator can be used for light of any wavelength.

3.7 ELLIPTICALLY AND CIRCULARLY POLARISED LIGHT

As we know, in plane polarized light the electric vector varies simple harmonically along a fixed straight line perpendicular to the direction of propagation. By the superposition of two plane polarized beam of monochromatic light under suitable conditions, the resultant light vector so produced rotates in a plane perpendicular to the direction of propagation.

If resultant light vectors magnitude remains constant during its rotation, its tip would trace out a *circle* and the resulting light is then said to be ***circularly polarized light***. However, the light vector either can rotate in clockwise or anticlockwise direction. Accordingly, such a wave is known as; left circularly or right circularly polarized light, respectively.

If resultant light vectors magnitude varies periodically between maximum and minimum values during its rotation, its tip would trace out an *ellipse* and the resulting light is then said to be

elliptically polarized light. However, the light vector either can rotate in clockwise or anticlockwise direction. Accordingly, such a wave is known as; left elliptically or right elliptically polarized light, respectively.

(a) **Superposition of two plane polarized light:**

Suppose that plane polarized light from a Nicol prism, as in Figure 16a, is incident normally on a thin plate of uniaxial crystal cut with faces parallel to the optic axis. Let A be the amplitude of vibration PA in the incident light. Upon entering the crystal, the vibrations will make some angle (say θ) with optic axis (Figure 16b), therefore the light will break up into two components, $A\cos\theta$ (along PE as *e-ray*) and $A\sin\theta$ (along PO as *o-ray*). The component $A\cos\theta$ having vibrations parallel to optic axis and the component $A\sin\theta$ having vibrations perpendicular to the optic axis. Now, the speed of *o-ray* is different from *e-ray*, therefore *o-ray* emerges from the uniaxial plate with a phase difference $\delta$with *e-ray*.

$$x = A\cos\theta\sin\omega t \qquad \text{and,} \qquad y = A\sin\theta\sin(\omega t+\delta)$$

Let, $a = A\cos\theta$, and $b = A\sin\theta$. Then, we have

$$x = a\sin\omega t \qquad \text{and,} \qquad y = b\sin(\omega t+\delta)$$

where, a and b are the amplitudes and δ is the phase difference between the two components. For the purpose of combining the two expressions, it can be written as

$$\frac{x}{a} = \sin\omega t$$

$$\frac{y}{b} = \sin(\omega t+\delta) = \sin\omega t\cos\delta + \cos\omega t\sin\delta$$

$$\frac{y}{b} = \frac{x}{a}\cos\delta + \sqrt{1-\frac{x^2}{a^2}}\sin\delta$$

$$\frac{y}{b} - \frac{x}{a}\cos\delta = \sqrt{1-\frac{x^2}{a^2}}\sin\delta$$

After squaring and rearranging the terms, we get

$$\frac{x^2}{a^2} + \frac{y^2}{b^2}\cos^2\delta - \frac{2xy}{ab}\cos\delta = \sin^2\delta - \frac{x^2}{a^2}\sin^2\delta$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab}\cos\delta = \sin^2\delta$$

This is the general equation of an *ellipse* in *x-y* plane whose axes are inclined to the coordinate axes. Hence, the path followed by the tip of resultant light vector, in general, an ellipse and the light emerging from the crystal plate, in general, ***elliptically polarized***. Some special cases of the phase difference are given below.

(i) **When $\boldsymbol{\delta = 0, 2\pi, 4\pi}$**, …, then $\cos\delta = 1$ and $\sin\delta = 0$.

Thus, general equation reduces to equation of *straight lines* passing through origin,

$$\left(\frac{y}{b} - \frac{x}{a}\right)^2 = 0$$

$$y = \frac{b}{a}x$$

with a positive slope b/a, and lying in the first and third quadrants of the *x-y* plane. The motion of the tip of resultant light vector is rectilinear and takes place along a diagonal of rectangle (Figure 17a). It means that the emergent light is ***linearly*** or ***plane polarized light***.

(ii) **When** $\boldsymbol{\delta} = \frac{\boldsymbol{\pi}}{\mathbf{2}}, \frac{\mathbf{5\pi}}{\mathbf{2}}, \frac{\mathbf{9\pi}}{\mathbf{2}}, \ldots$, then $\cos\delta = 0$ and $\sin\delta = 1$.

Thus, general equation reduces to the equation of an ellipse,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

whose principle axes lie along the *x* and *y* axes, as in Figure 18a. The tip of resultant light vector moves in an elliptical path in the *anti-clockwise* direction. Therefore, the emergent light is called ***left-handed elliptically polarized light***.

In case, $\theta = \pi/4$, then $a = b = a/\sqrt{2}$. Thus, general equation reduces to the equation of a circle of radius $a/\sqrt{2}$.

$$x^2 + y^2 = a^2/2$$

The tip of resultant light vector moves in a circular path in the *anti-clockwise* direction, as in Figure 19a. Therefore, the emergent light is called ***left-handed circularly polarized light***.

(iii) **When** $\boldsymbol{\delta} = \boldsymbol{\pi}, \mathbf{3}\boldsymbol{\pi}, \mathbf{5}\boldsymbol{\pi}, \ldots$, then $\cos\delta = -1$ and $\sin\delta = 0$.

Thus, general equation reduces to equation of *straight lines* passing through origin,

$$\left(\frac{y}{b} + \frac{x}{a}\right)^2 = 0$$

$$y = -\frac{b}{a}x$$

with a negative slope b/a, and lying in the second and forth quadrants of the *x-y* plane. The motion of the tip of resultant light vector is rectilinear and takes place along a diagonal of rectangle (Figure 17b). It means that the emergent light is ***linearly*** or ***plane polarized light***.

(iv) **When** $\boldsymbol{\delta} = \frac{\mathbf{3\pi}}{\mathbf{2}}, \frac{\mathbf{7\pi}}{\mathbf{2}}, \frac{\mathbf{11\pi}}{\mathbf{2}}, \ldots$, then $\cos\delta = 0$ and $\sin\delta = 1$.

Thus, general equation reduces to the equation of an ellipse,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

whose principle axes lie along the *x* and *y* axes, as in Figure 18b. The tip of resultant light vector moves in an elliptical path in the *clockwise* direction. Therefore, the emergent light is called ***right-handed elliptically polarized light***.

In case, $\theta = \pi/4$, then $a = b = a/\sqrt{2}$. Thus, general equation reduces to the equation of a circle of radius $a/\sqrt{2}$.

$$x^2 + y^2 = a^2/2$$

The tip of resultant light vector moves in a circular path in the *clockwise* direction, as in Figure 19b. Therefore, the emergent light is called ***right-handed circularly polarized light***.

Thus, we conclude that the plane polarized and circularly polarized lights are the special cases of elliptically polarized light.

(b) Production of Elliptically Polarized Light:

If we allow a monochromatic beam of plane polarized light to be incident normally on a quarter wave plate, in such a way that the direction of vibrations in the incident light being inclined at an angle other than 0, 45 or 90, (i.e. $\theta \neq 0, \frac{\pi}{4}, \frac{\pi}{2}$) to the direction of optic axis. On entering both *e-ray* and *o-ray* are in the same phase but on emergence there will be a phase difference of $\delta = \frac{\pi}{2}$, (or path difference of $\lambda/4$) between both the rays. Since, $\theta \neq \frac{\pi}{4}$, the amplitudes of *e-ray* and *o-ray* are not equal. Hence, on emergence from the quarter wave plate the emergent light will be elliptically polarized, whose magnitude varies periodically between maximum and minimum of its amplitude. If quarter wave plate is made of negative crystal the elliptically polarized light will be *left handed* and for positive crystal that will be *right handed*.

(c) Production of Circularly Polarized Light:

If we allow a monochromatic beam of plane polarized light to be incident normally on a quarter wave plate, in such a way that the direction of vibrations in the incident light being inclined at an angle 45 (i.e. $\theta = \frac{\pi}{4}$) to the direction of optic axis. On entering both *e-ray* and *o-ray* are in the same phase but on emergence there will be a phase difference of $\delta = \frac{\pi}{2}$, (or path difference of $\lambda/4$) between both the rays. Since, $\theta = \frac{\pi}{4}$, the amplitudes of *e-ray* and *o-ray* are equal. Hence, on emergence from the quarter wave plate the emergent light will be circularly polarized, whose magnitude remain constant in its amplitude. If quarter wave plate is made of negative crystal the circularly polarized light will be *left handed* and for positive crystal that will be *right handed*.

(d) Analysis of Polarized Light:

A plane wave can have different states of polarizations, which may be any one of the following, but for naked eye all the states will appear to be same.

i. Unpolarized Light – UPL
ii. Plane Polarized Light – PPL
iii. Circularly Polarized Light – CPL
iv. Elliptically Polarized Light – EPL
v. Mixture of Unpolarized and Plane Polarized Light – UPL and PPL
vi. Mixture of Unpolarized and Circularly Polarized Light – UPL and CPL
vii. Mixture of Unpolarized and Elliptically Polarized Light – UPL and EPL

STEP I: To determine the state of polarization of light beam, we introduce a Nicol prism in the path of the beam and rotate about the direction of propagation (as shown in Figure 20). Then any of the following three possibilities can occur.