Radius of the rings: Let P and P' are two points on the screen, such that PP' = x. The path difference between the secondary waves from A and B reaching P' can be given by

$$\Delta = \mathrm{BP}' - \mathrm{AP}'$$

$$\Delta = [(x+r)^2 + b^2]^{1/2} - [(x-r)^2 + b^2]^{1/2}$$

$$\Delta \cong b\left[1 + \frac{(x+r)^2}{2b^2}\right] - b\left[1 + \frac{(x-r)^2}{2b^2}\right]$$

$$\Delta = \frac{1}{2b}[(x+r)^2 - (x-r)^2]$$

$$\Delta = \frac{2rx}{b}$$

The point P' will be of minimum or maximum intensity corresponding to the path difference is an even or odd multiple of $\lambda/2$.

For dark rings,

$$\Delta = \frac{2rx_m}{b} = m\lambda$$

Here x_m represents the radius of m^{th} dark ring. The radii of the dark rings are given by,

$$x_m = m\frac{b\lambda}{2r}$$

For bright rings,

$$\Delta = \frac{2rx_m}{b} = \left(m + \frac{1}{2}\right)\lambda$$

Here x_m represents the radius of m^{th} bright ring. The radii of the bright rings are given by,

$$x_m = \left(m + \frac{1}{2}\right)\frac{b\lambda}{2r}$$

The diffraction pattern so obtained consists of a central disc (bright or dark) surrounded by concentric alternate dark and bright rings of gradually decreasing intensity (Figure 20).

4.4 RESOLVING POWER

We have seen that when light from point source passes through any obstacle or aperture, the image of such point source is not a point but has a definite size and is surrounded by a diffraction pattern (as in Figure 20). If there are two point sources very close together the two diffraction pattern (each due to each point source) thus formed and they may overlap. Hence, it may be difficult to distinguish them as separate diffraction pattern. Therefore, it is necessary to know the capability of any optical instrument, such as telescope or microscope, that how far it is able to distinguish the overlapping diffraction pattern due to two point sources.

The ability of an optical instrument to just resolve the image of two nearby point sources is called its ***resolving power****.*

In case of prism or grating spectrometer, we are concerned with two spectral lines close to each other and sometimes it could be difficult to distinguish them. Therefore, *the* ***resolving power*** *of such an optical instrument is define as its ability to just resolve these two close spectral lines.*

An optical system is said to be able to resolve two point objects if the corresponding diffraction pattern are sufficiently small or sufficiently separated to be distinguish as separate image.

4.4.1 Rayleigh Criterion of Resolution

For a detailed study of the resultant intensity distribution in the diffraction pattern of closely spaced point sources, Lord Rayleigh arrived at the conclusion that, *the two equally bright sources or spectral lines of equal intensity could be just resolved by any optical system, when their distance apart is such that the central maximum in the diffraction pattern due to one source coincided exactly with the first minimum in the diffraction pattern due to the other*. This is known as the ***Rayleigh's criterion of resolution***.

The physical significance of this can be explained by considering the diffraction pattern produced by two wavelengths λ and λ+dλ. Let the dλ be such that the principal maximum of one coincides exactly over second minimum of the other (Figure 21). In this case there is a distinct point of zero intensity in the middle of resultant intensity curve which has been obtained by summing the intensities due to separate patterns. Thus, the two spectral lines are ***distinctly separated***.

However, if dλ is smaller than there is one limiting value for which the angular separation between their principal maxima is such that the principal maximum of one falls exactly over the first minimum due to other (Figure 22). Therefore, the resultant intensity curve is a double humped curve with a distinct dip between two principal maxima and equal to those of original maxima. The two spectral lines, under this condition, are said to be ***just resolved***.

If dλ is smaller than this limiting value than the two spectral lines come still closer and the intensity curve of the principal maxima show considerable overlapping. The resultant intensity curve exhibits no dip but only one maximum in centre, thereby indicating as if it is a diffraction pattern of only one spectral line. Thus, under this condition, the two spectral lines are ***not resolved*** in their diffraction pattern.

4.4.2 Resolving Power of Diffraction Grating

In the case of a diffraction grating the resolving power (RP) refers to the power of distinguishing two nearby spectral lines and is defined by the following equation

$$RP = \frac{\lambda}{d\lambda}$$

where $d\lambda$ is the separation of two wavelengths which the grating can just resolve. The smaller the value of $d\lambda$, the larger the resolving power (see the Figure 23).

The Rayleigh criterion can be used to define the limit of resolution. According to this criterion, if the principal maximum corresponding to the wavelength λ+$d\lambda$ coincides exactly on the first minimum of the wavelength λ (on the either side of the principal maximum), then the two wavelengths λ and λ+$d\lambda$ are said to be just resolve. If the point of such coincide subtend a common diffraction angle θ and if we are observing at the m^{th} order spectrum, then the two wavelengths λ and λ+$d\lambda$ will be just resolved if the principal maximum of λ+$d\lambda$ is such that

$$d\sin\theta = m(\lambda + d\lambda)$$

and the first minimum of λ satisfy the following

$$d\sin\theta = m\lambda + \frac{\lambda}{N}$$

This is because the extra path difference of λ/N will introduced for first minimum. Thus, by equating both the equation, we can get the ***resolving power*** of diffraction grating

$$RP = \frac{\lambda}{d\lambda} = mN$$

Thus, the resolving power of a grating is independent of the grating element, d, but is directly proportional to the order of spectrum, m and the total number of lines, N on the grating surface.

As $\theta + d\theta$ is another common diffraction angle at just resolution. Then the first minimum of λ+$d\lambda$ wave will have the condition

$$d\sin(\theta + d\theta) = m\lambda + \frac{\lambda}{N}$$

Here, terms with smaller magnitude ($md\lambda$ and $d\lambda/N$) has been neglected. Since

$$\sin(\theta + d\theta) = \sin\theta\cos d\theta + \cos\theta\sin d\theta \cong \sin\theta + d\theta\cos\theta$$

Because principal maxima of λ corresponds to, $d\sin\theta = m\lambda$. Therefore, we have

$$d\theta = \frac{\lambda}{Nd\cos\theta}$$

Then, the ***dispersive power*** ω of a diffraction grating will be

$$\omega = \frac{d\theta}{d\lambda} = \frac{\lambda}{Nd\cos\theta\, d\lambda} = \frac{m}{d\cos\theta} = \frac{mN'}{\cos\theta}$$

where N' is the number of lines per centimeter and N is the total number of lines on grating.

It is obvious from the above equation that the dispersive power depends on number of lines per cm whereas the resolving power on the total number of lines on grating surface.

High dispersive power refers to wide separation of the spectral lines whereas high resolving power refers to the ability of the instrument to show nearby spectral lines as separates ones.

4.4.3 Resolving Power of Prism

The resolving power of a prism represents its ability to just resolve the spectral lines of two wavelengths very close together. Each spectral line in the spectrum formed by a prism consists of a diffraction pattern with central maximum having on either side of it alternate minima and maxima. If λ and $\lambda + d\lambda$ are two very close wavelengths which can be just resolved with a prism, then the ratio $\lambda/d\lambda$ is a measure of the resolving power of the prism.

Let a parallel beam of light through collimator consists of wavelengths λ and λ + dλ be reflected through a prism which is placed in the position of minimum deviation. Suppose, BP is the incident plane wavefront, CQ and CQ' are the emergent wavefronts corresponding to wavelengths λ and λ + dλ, respectively and P_1 and P_2 are the position of the central maxima of diffraction pattern due to wavelengths λ and λ + dλ, respectively. Here, L is the objective lens of telescope of spectrometer (Figure 24).

As the emergent beam has a rectangular cross-section, the prism may be considered as a rectangular aperture of width a, which is equal to the width of the emergent beam. The angle $d\theta_1$ between central maximum and first minimum λ of due to a rectangular aperture of width a is given by

$$a \sin d\theta_1 = \lambda$$

$$d\theta_1 \cong \frac{\lambda}{a}$$

as $d\theta_1$ is small.

For just resolution the position of central maximum of one spectral line of wavelength $\lambda + d\lambda$ will lie to the first minimum of other spectral line of wavelengthλ.Thus, the angle between them must be equal to the angle between central maximum and first minimum of any one. Thus, for just resolution,

$$d\delta = d\theta_1 = \frac{\lambda}{a}$$
$$a\, d\delta = \lambda$$

From the Figure 24,

$$\alpha + A + \alpha + \delta = \pi$$

$$\alpha = \left[\left(\frac{\pi}{2}\right) - \left(\frac{A+\delta}{2}\right)\right]$$

$$\sin\alpha = \sin\left[\left(\frac{\pi}{2}\right) - \left(\frac{A+\delta}{2}\right)\right] = \cos\left(\frac{A+\delta}{2}\right)$$

But, from Figure 24

$$\sin\alpha = \frac{a}{l}$$

$$\sin\frac{A}{2} = \frac{t}{2l}$$

In the case of prism,

$$\mu = \frac{\sin\left(\frac{A+\delta}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$$\sin\left(\frac{A+\delta}{2}\right) = \mu\sin\left(\frac{A}{2}\right)$$

Differentiating with respect to λ, we have

$$\frac{1}{2}\cos\left(\frac{A+\delta}{2}\right)\frac{d\delta}{d\lambda} = \frac{d\mu}{d\lambda}\sin\left(\frac{A}{2}\right)$$

$$\frac{1}{2}\left(\frac{a}{l}\right)\frac{d\delta}{d\lambda} = \frac{d\mu}{d\lambda}\left(\frac{t}{2l}\right)$$

$$a\frac{d\delta}{d\lambda} = t\frac{d\mu}{d\lambda}$$

The ***resolving power*** of prism is, therefore

$$RP = \frac{\lambda}{d\lambda} = t\frac{d\mu}{d\lambda}$$

Thus, the resolving power of a prism is directly proportional to the width of the base of the prism and also to the rate of change of refractive index of the material of the prism with the wavelength.

The ***dispersive power*** ω of a prism can be derived as,

$$\omega = \frac{d\theta}{d\lambda} = \frac{d\delta}{d\lambda} = \frac{t}{a}\frac{d\mu}{d\lambda}$$

This indicates that the dispersion of prism depends on the material of prism. Therefore, the relative spacing of the spectral lines varies with the material.

4.4.4 Resolving Power of Telescope

It is its ability to form separate image of two distant point objects situated close to each other. It is measured in terms of the angle subtended at its objective by two nearby distant objects whose images are just resolved by the objective of the telescope. This angle is called the ***limit of resolution*** of the telescope and reciprocal of this angle is called the ***resolving power*** of the telescope. Thus, the smaller the value of this angle the higher is the resolving power of the telescope (Figure 25).

Let AB be the objective of the telescope and O the distant object sending monochromatic light of wavelength λ. The objective lens of the telescope acts like a ***circular aperture*** and hence the image of O is a diffraction pattern consists of a central bright disc with centre at P surrounded by alternate dark and bright rings. The image of another point object O', situated close to O, is also similar diffraction patterns with its centre at P'. These diffraction pattern overlap and two point objects can be just resolved, when according to Rayleigh criterion the separation PP' between the centres of two central maxima is at least equal to the radius of either one i.e. x_1.

The radius of the central disc is given by

$$x_1 = \frac{b\lambda}{2r} = \frac{b\lambda}{D}$$

where D is the diameter of the aperture.

The angle subtended by the central disc at the centre of the aperture

$$\sin\theta = \frac{x_1}{b} = \frac{\lambda}{D}$$

As shown in Figure 25 this angle θ is equal to the angle α subtended at its objective by two nearby distant objects, whose images are just resolved by the objective of telescope. In practice, $D \gg \lambda$, therefore θ is very small and hence we may put $\sin\theta \cong \theta$. Therefore, we have

$$\theta = \frac{\lambda}{D}$$

However, in case of circular aperture, Airy has shown that the ***limit of resolution*** of the telescope is more correctly can be written by multiplying it with a coefficient 1.22. Hence,

$$\theta = \frac{1.22\,\lambda}{D}$$

Hence, the ***resolving power*** of the telescope,

$$RP = \frac{1}{\theta} = \frac{D}{1.22\,\lambda}$$

If f is the focal length of the telescope objective, then the radius of the first dark ring of central bright disc ring of central bright disc called the Airy's disc is given by

$$x_1 = f\theta = \frac{1.22\,\lambda}{D}$$

From this equation, if the focal length of the objective is small, the wavelength is small and the aperture is large, then the radius of the central bright disc is small. The diffraction pattern will appear sharper and the angular separation between two just resolvable point objects will be smaller. Hence, the resolving power of the telescope will be higher.