4.1 DIFFRACTION

Suppose a light wave incident on a slit AB of sufficient width b, as shown in Figure 1. According to *concept of rectilinear propagation of light* the region A'B' on the screen should be uniformly illuminated and remaining portion (above A' and below B') should be absolutely dark (known as *geometrical shadow region*).

However, if the width b of the slit is reduced, so that it becomes of the order of wavelength of incident light, then the light intensity in the region A'B' is not uniform. Further, there is some light intensity which spreads out to some extent into the geometrical shadow region of the screen. If the width of the slit is made smaller further, larger amount of intensity spreads into the shadow region. This is known as the phenomena of ***diffraction*** and the intensity distribution on the screen is known as the *diffraction pattern*.

The bending of light round the corners and spreading out of light intensity into the region of geometrical shadow when it passes through a narrow opening (such as narrow hole, slit, aperture, edges etc.) is known as the ***diffraction***.

4.1.1 Difference between Interference and Diffraction

Interference	Diffraction
• Due to superposition between two wavefronts originating from two separated coherent sources.	• Due to superposition of secondary wavelets originating from different points of the same wavefront.
• Interference is achieved through two ways; division of wavefront and division of amplitude.	• Diffraction is achieved by two ways; Fraunhofer diffraction and Fresnel diffraction.
• All maxima are of same intensity.	• Maxima are of varying intensity.
• Fringe widths are equal or regular.	• Fringe widths are not equal or regular.

4.1.2 Classification Of Diffraction Phenomena

Diffraction phenomena are conveniently divided into two classes;

(i) Fraunhofer Class of Diffraction
(ii) Fresnel Class of Diffraction

Those in which the source of light and the screen on which the pattern is observed are effectively at infinite distances from the aperture are called ***Fraunhofer class of diffraction***.

Those in which either the source of light or the screen or both are at finite distance from the aperture are called ***Fresnel's class of diffraction***.

Fraunhofer diffraction is much simpler to achieve and can be observed by sending the light parallel with a lens and focusing it on a screen with another lens placed behind the aperture which effectively fulfill the condition that the source and screen should be at infinity. On the other hand, no lenses are necessary in the observation of Fresnel's diffraction, but the wavefronts are divergent instead of plane and therefore more complex to treat mathematically.

4.1.3 Difference between Fraunhofer and Fresnel Diffraction

Fraunhofer Diffraction	Fresnel Diffraction
• The source and screen are at infinite distance from the diffraction aperture.	• The source and screen are at finite distance from the diffraction aperture.
• The wavefronts are plane which is achieved by using convex lens.	• The wavefronts are divergent, either spherical or cylindrical.
• Diffracted rays are focused on screen by a convex lens for observation.	• No lens or mirrors are required for the observation.
• Slits are used to obtain diffraction.	• Zone plate is required to obtain diffraction.
• The centre of the diffraction pattern is always bright.	• The centre of the diffraction pattern may be bright or dark.

4.2 FRAUNHOFER DIFFERACTION

4.2.1 SINGLE SLIT

A slit is a rectangular aperture of length large compared to its breadth. Consider a slit S to be set up as shown in Figure 2 and to be illuminated by parallel monochromatic light from the narrow slit S' at the principal focus of the lens L_1. The light focused by another lens L_2 on a screen P at its focus will form a diffraction pattern. The explanation of the single slit pattern lies in the interference of the secondary wavelets which can be assumed as spread out from every point on the wavefront at the instant that it strikes the slit.

Figure 3 represents a section of a slit of width *b*. Let *ds* be an element of width of the wavefront in the plane of the slit, at a distance *s* from the centre O. The parts of each secondary wave which travels normal to the plane of the slit will be focused at P_o, while those which travel at any angle θ will reach at P.

Considering first the wavelet emitted by the element *ds* situated at the origin O, its amplitude will be directly proportional to the length *ds* and inversely proportional to the distance *x*. At P, it will produce an infinitesimal displacement which may be expressed as,

$$dy_o = \frac{ads}{x}\sin(\omega t - kx)$$

As the position of *ds* is varied, the displacement it produces will vary in phase because of the different path length to P. When it is at a distance *s below the origin* the contribution will be,

$$dy_s = \frac{ads}{x}\sin\big(\omega t - k(x+\Delta)\big)$$
$$dy_s = \frac{ads}{x}\sin(\omega t - kx - ks\sin\theta)$$

and at a distance *s above the origin* the contribution will be

$$dy_{-s} = \frac{ads}{x}\sin(\omega t - kx + ks\sin\theta)$$

The contribution from *pairs of elements* symmetrically placed at s and $-s$ being,

$$dy = dy_s + dy_{-s}$$
$$dy = \frac{ads}{x}[\sin(\omega t - kx - ks\sin\theta) + \sin(\omega t - kx + ks\sin\theta)]$$
$$dy = \frac{ads}{x}[2\cos(ks\sin\theta)\sin(\omega t - kx)]$$

which must be integrated from $s = 0$ to $b/2$. In doing so, x may be regarded as constant, insofar as it affects the amplitude. Thus,

$$y = \frac{2a}{x}\sin(\omega t - kx)\int_0^{b/2}\cos(ks\sin\theta)ds$$
$$y = \frac{2a}{x}\left[\frac{\sin(ks\sin\theta)}{k\sin\theta}\right]_0^{b/2}\sin(\omega t - kx)$$
$$y = \frac{ab}{x}\frac{\sin\left(\frac{1}{2}kb\sin\theta\right)}{\frac{1}{2}kb\sin\theta}\sin(\omega t - kx)$$
$$y = A_o\frac{\sin\beta}{\beta}\sin(\omega t - kx)$$

The resultant vibration will therefore be a simple harmonic one, the amplitude of which varies with the position of P. We may represent its amplitude as

$$A = A_o\frac{\sin\beta}{\beta}$$

where $\beta = \frac{1}{2}kb\sin\theta = \frac{\pi}{\lambda}b\sin\theta$, and $A_o = \frac{ab}{x}$. The intensity on the screen is then

$$I = I_o\frac{\sin^2\beta}{\beta}$$

It is obvious from above equation that the in intensity is zero when $\beta = m\pi$ *i.e.*,

$$b\sin\theta = m\lambda \qquad \text{where } m = \pm1, \pm2, \pm3, \ldots \qquad \text{but } m \neq 0$$

as the condition for ***minima***. The first minima occurs at $\theta = \pm\sin^{-1}\frac{\lambda}{b}$; the second minima at $\theta = \pm\sin^{-1}\frac{2\lambda}{b}$; and so on.

For $m = 0$ i.e., at $\beta = 0$ the equation will be indeterminate. Using L' Hospital rule, the intensity will be maximum, i.e. $I = I_o$ at $\beta = \theta = 0$. This is the ***central maxima***.

In order to determine the position of ***secondary maxima***, we differentiate the equation with respect to β and set it equal to zero. Thus,

$$\frac{dI}{d\beta} = I_o\left[\frac{2\sin\beta\cos\beta}{\beta^2} - \frac{2\sin^2\beta}{\beta^2}\right] = 0$$

$$\sin\beta\,[\beta - \tan\beta] = 0$$

The condition $\sin\beta = 0$, or $\beta = m\pi$ corresponds to minima. The conditions for maxima are roots of the following equations,

$$\tan\beta = \beta$$

The root $\beta = 0$ corresponds to the central maximum. The other roots can be found by determining the points of intersections of the curves $y = \beta$ and $y = \tan\beta$. The intersections occur at $\beta = 1.43\pi, \beta = 2.46\pi, \beta = 3.47\pi$, etc. is known as the first maxima, second maxima, third maxima, etc. (see the resultant intensity distribution in the Figure 4).

Since, $[\sin(1.43\pi)/1.43\pi]^2$ is about 0.0496, the intensity of the first maxima is about 4.96% of the central maxima. Similarly, the intensities of the second and third maxima are about 1.68% and 0.83% of the central maxima, respectively (as shown in Figure 4).

The intensities of the secondary maxima can be calculated to a very close approximation by finding the values of $\sin^2\beta/\beta$ at the half way positions i.e., where $\beta = 3\pi/2, 5\pi/2, 7\pi/2, \ldots\ldots\ldots$. This gives $4/9\pi^2, 4/25\pi^2, 4/45\pi^2, \ldots\ldots\ldots$ or $1/22.2, 1/61.7, 1/121, \ldots\ldots\ldots$ of the intensity of the central maxima.

4.2.2 N-SLITS

If we study the Fraunhofer diffraction pattern produced by N-parallel slits, each of width b, separated by an opaque space of width c, the distance between centers of the slit is $d = b + c$, (as shown in Figure 5). At the end of this discussion, we will find that the resultant intensity distribution is a product of the single slit diffraction pattern and the interference pattern produced by N-slits separated by a distance d.

In order to calculate the diffraction pattern, we use a method similar to that used for the case of single slit. If the diffracted rays make an angle θ with the normal to the slits (as shown in Figure 6), then $\Delta = d\sin\theta$ represents the path difference between the disturbances reaching the point P from two corresponding points on the consecutive slits which are separated by a distance d. Then, the displacement produced by the slits at the point P will, therefore be given by the equation similar to the single slit equation, i.e.

$$y_1 = A_o\frac{\sin\beta}{\beta}\sin(\omega t - kx)$$
$$y_2 = A_o\frac{\sin\beta}{\beta}\sin(\omega t - kx - kd\sin\theta)$$
$$y_3 = A_o\frac{\sin\beta}{\beta}\sin(\omega t - kx - 2kd\sin\theta)$$
$$\vdots$$
$$y_N = A_o\frac{\sin\beta}{\beta}\sin(\omega t - kx - (N-1)kd\sin\theta)$$

Thus, the resultant displacement will essentially be a sum of N terms,

$$y_N = y_1 + y_2 + y_3 + \ldots + y_N$$

$$y_N = A_o \frac{\sin\beta}{\beta}[\sin(\omega t - kx) + \sin(\omega t - kx - kd\sin\theta) + \sin(\omega t - kx - 2kd\sin\theta) + \cdots + \sin(\omega t - kx - (N-1)kd\sin\theta)]$$

Using trigonometric identity,

$$\sin\alpha + \sin(\alpha+\beta) + \sin(\alpha+2\beta) + \cdots + \sin(\alpha+(N-1)\beta) = \frac{\sin\frac{1}{2}N\beta}{\sin\frac{1}{2}\beta}\sin\left[\alpha + \frac{1}{2}(N-1)\beta\right]$$

Thus, equation reduces to

$$y_N = A_o \frac{\sin\beta}{\beta}\frac{\sin N\gamma}{\sin\gamma}\sin(\omega t - kx - (N-1)\gamma)$$

$$y_N = A\sin(\omega t - kx - (N-1)\gamma)$$

where, $\gamma = \frac{1}{2}kd\sin\theta = \frac{\pi}{\lambda}kd\sin\theta$, and the amplitude of the resultant displacement

$$A = A_o\frac{\sin\beta}{\beta}\frac{\sin N\gamma}{\sin\gamma}$$

The corresponding intensity distribution will be,

$$I = I_o\frac{\sin^2\beta}{\beta^2}\frac{\sin^2 N\gamma}{\sin^2\gamma}$$

Thus, the intensity distribution is a product of two terms; the first term $(\sin^2\beta/\beta^2)$ represents the diffraction pattern produced by a single slit and the second term $(\sin^2 N\gamma/\sin^2\gamma)$ represents the interference pattern produced by N equally spaced point sources.

The new factor $(\sin^2 N\gamma/\sin^2\gamma)$ attains maximum values equal to N^2 for $\gamma = 0, \pi, 2\pi, \ldots$. Although the quotient becomes indeterminate at these values, this result can be obtained by using L' Hospital rule.

$$\lim_{\gamma\to 0}\frac{\sin N\gamma}{\sin\gamma} = \lim_{\gamma\to 0}\frac{N\cos N\gamma}{\cos\gamma} = \pm N$$

Thus, the resultant amplitude and the corresponding intensity distributions are given by,

$$A = NA_o\frac{\sin\beta}{\beta}$$

$$I = N^2 I_o\frac{\sin^2\beta}{\beta^2}$$

These maxima correspond the position for the $\gamma = m\pi$, i.e. when

$$d\sin\theta = m\lambda$$

$$(b+c)\sin\theta = m\lambda$$

where $m = 0,\ \pm1,\ \pm2,\ \pm3, \dots . \dots$ Such maxima are known as ***principal maxima***. They are more intense, in the ratio of the square of the number of slits N.

To find the minima of the function $(\sin^2 N\gamma / \sin^2 \gamma)$, we note that the numerator becomes zero more often than the denominator. This occurs at the values; $N\gamma = 0, \pi, 2\pi, \dots$ or $N\gamma = p\pi$, where $p = 0,\ \pm1,\ \pm2,\ \pm3, \dots . \dots$

But, in the case when $p = 0, N, 2N, \dots$ then $\gamma = 0, \pi, 2\pi, \dots$ So, for these values the denominator will also be zero. As we discuss above, this is the condition for principal maxima.

Hence, the condition for ***minima*** is,

$$N\gamma = p\pi \qquad \text{but } p \neq 0, N, 2N, \dots$$

$$d\sin\theta = \frac{\lambda}{N}, \frac{2\lambda}{N}, \frac{3\lambda}{N}, \dots, \dots, \frac{(N-1)\lambda}{N}, \frac{(N+1)\lambda}{N}, \dots, \dots, \frac{(2N-1)\lambda}{N}, \frac{(2N+1)\lambda}{N}, \dots, \dots$$

Note that values $0, N\lambda/N, 2N\lambda/N, \dots, \dots$ for which $d\sin\theta = m\lambda$; condition for principal maxima, are not included in the above series for minima.

Between two neighbouring principal maxima there will be $(N - 1)$ minima, point of *zero intensity*. Between two such consecutive minima, there has to be a maxima, known as ***secondary maxima***. For large value of N, the secondary maxima produced are of much smaller intensity than the principal maxima, thus can be neglected (see the Figure 7).

(a) **Missing Order:** The actual position of the principal maxima will occur at the angle where $d\sin\theta = m\lambda$, provided the variation of the diffraction term$(\sin^2\beta/\beta^2)$ is not large. A particular principal maxima may be absent, if this condition for a principal maxima of interference produced by N-slits and for a minima of the diffraction are both fulfilled for the same value of angle θ, i.e. for

$$d\sin\theta = m\lambda \qquad \text{where, } m = 0,\ \pm1,\ \pm2,\ \pm3, \dots . \dots$$

$$b\sin\theta = p\lambda \qquad \text{where, } p = 0,\ \pm1,\ \pm2,\ \pm3, \dots . \dots$$

So that,

$$\frac{d}{b} = \frac{m}{p}$$

This ratio determines the orders which are missing (shown in Figure 7), referred to ***missing order***.

(b) **Width of Principal Maxima:** We have seen above that in the diffraction pattern produced by N slits, the m^{th} order principal maxima occurs at

$$d\sin\theta_m = m\lambda \qquad \text{where, } m = 0,\ \pm1,\ \pm2,\ \pm3, \dots . \dots$$

Further, the minima occur at the angles given by

$$d\sin\theta_m = \frac{p\lambda}{N}$$

But $p \neq 0, N, 2N, \ldots.$ If $\theta_m + \Delta\theta_m$ and $\theta_m - \Delta\theta_m$ represents the angle of diffraction corresponding to the first minimum on either side of the principal maxima, then $\Delta\theta_m$ is known as the ***angular half width*** of the m^{th} order principal maxima (Figure 8). Clearly,

$$d\sin(\theta_m \pm \Delta\theta_m) = m\lambda \pm \frac{\lambda}{N}$$

Since, we know from trignometry

$$\sin(\theta_m \pm \Delta\theta_m) = \sin\theta_m \cos\Delta\theta_m \pm \cos\theta_m \sin\Delta\theta_m \cong \sin\theta_m \pm \Delta\theta_m \cos\theta_m$$

Hence,

$$d\sin\theta_m \pm \Delta\theta_m d\cos\theta_m = m\lambda \pm \frac{\lambda}{N}$$

Thus, angular half width $\Delta\theta_m$ of the m^{th} order principal maxima is

$$\Delta\theta_m \cong \frac{\lambda}{Nd\cos\theta_m}$$

This shows that the principal maxima become sharper as N increase.

4.2.3 DIFFRACTION GRATING

An arrangement which essentially consists of a large number of equidistant slits of the same width is known as a ***diffraction grating***, the corresponding diffraction pattern is known as the *grating spectrum*. Since the exact position of the principal maxima in the diffraction pattern depend on the wavelength, the principal maxima corresponding to different spectral lines (colour or wavelength) will correspond to different angles of diffraction. Thus, the grating spectrum provides us with an easily obtainable experimental setup for determination of wavelengths.

A good quality grating therefore requires a large number of slits, about more than 15,000 LPI (lines per inch). This is achieved by ruling grooves with a diamond point on an optically transparent sheet of material; the grooves act as opaque spaces.

In *transmission gratings*, glass is used. The lines act as opaque while the spaces between them transmit light, act as slits. In *reflection gratings*, the lines ruled on metal are opaque but the space between two such ruled lines reflect light, regularly.

Commercially available gratings are produced by taking the cast of an actual grating on a transparent film like that of cellulose acetate. Diffraction gratings are used to produce very sharp spectra and for measuring wavelengths accurately. They have replaced the prism in modern spectroscopy.

We have shown that the positions of the principal maxima are given by