

INTERFERENCE

$$2d \cos \theta' = m\lambda \quad \text{bright} \quad \textbf{constructive interference}$$

$$2d \cos \theta' = \left(m + \frac{1}{2}\right)\lambda \quad \text{dark} \quad \textbf{destructive interference}$$

and the condition for interference by reflected light beam is given by,

$$2d \cos \theta' = m\lambda \quad \text{dark} \quad \textbf{destructive interference}$$

$$2d \cos \theta' = \left(m + \frac{1}{2}\right)\lambda \quad \text{bright} \quad \textbf{constructive interference}$$

Thus, the conditions for constructive or destructive interference are just opposite in reflected beam than the transmitted beam. It is because reflected light beams suffer odd numbers of times reflection during multiple reflection which keeps them always out of phase by π , while transmitted lights suffer even number of times reflection so the beams are always in phase to each other.

This is because the central spot of transmitted light circular fringes is bright whereas of reflected light, it is dark. Thus, the two patterns so obtained are **complementary** to each other.

(c) Theory: Let us consider a plane parallel film of thickness d and of refractive index n , enclosed between plates of refractive index n' , illuminated by a plane wave at a finite angle. If δ represents the phase difference between two successive waves emanating from the film, we have

$$\delta = \frac{2\pi}{\lambda} \Delta$$

$$\delta = \frac{4\pi n d \cos \theta'}{\lambda}$$

Let t and r represent the amplitude transmission coefficient and amplitude reflection coefficient (or transmittance and reflectance), respectively; when the waves are travelling from glass plate to air film and t' and r' represent the corresponding quantities, when the waves are travelling from air film to glass plate (see Figure 12).

We know from Stoke's relations; $r = -r'$, and $1 - tt' = r^2 = r'^2$. If R and T represents the *energy reflectivity* and *energy transmittivity*, respectively of the film surface, then it can be shown that; $R = r^2$, and $T = tt'$. Thus,

$$R + T = 1$$

The R and T are usually referred to as *reflectivity* and *transmittivity*, respectively.

Now suppose a is the amplitude of the incident light, then the complex amplitudes of the successive reflected waves will be

$$ar, atr't'e^{i\delta}, atr'^3t'e^{2i\delta}, atr'^5t'e^{3i\delta}, \dots \dots \dots$$

and the complex amplitudes of successive transmitted waves will be

$$att', att'r'^2e^{i\delta}, att'r'^4e^{2i\delta}, att'r'^6e^{3i\delta}, \dots \dots \dots$$

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For the fringe system formed by the reflected light, the sum of complex amplitudes is,

$$A_r e^{i\theta} = ar + atr't'e^{i\delta} + atr'^3t'e^{2i\delta} + atr'^5t'e^{3i\delta} + \dots$$

$$= a[r + r'tt'e^{i\delta}(1 + r'^2e^{i\delta} + r'^4e^{2i\delta} + \dots)] = a\left[r - rTe^{i\delta} \cdot \frac{1}{1 - Re^{i\delta}}\right]$$

$$A_r e^{i\theta} = ar \left[\frac{1 - e^{i\delta}}{1 - Re^{i\delta}} \right]$$

For the fringe system formed by the transmitted light, the sum of complex amplitudes is,

$$A_t e^{i\theta} = att' + att'r'^2e^{i\delta} + att'r'^4e^{2i\delta} + att'r'^6e^{3i\delta} + \dots$$

$$= aT[(1 + r'^2e^{i\delta} + r'^4e^{2i\delta} + \dots)]$$

$$A_t e^{i\theta} = aT \left[\frac{1}{1 - Re^{i\delta}} \right]$$

The intensity is the product of the quantity by its complex conjugate, which yields the intensity of reflected system

$$I_r = I_0 \cdot r \left[\frac{1 - e^{i\delta}}{1 - Re^{i\delta}} \right] \cdot r \left[\frac{1 - e^{-i\delta}}{1 - Re^{-i\delta}} \right]$$

Because, $\frac{I_r}{I_0} = \left(\frac{A_r}{a}\right)^2$, so

$$I_r = I_0 R \frac{2 - 2 \cos \delta}{1 + R^2 - 2R \cos \delta} = I_0 \frac{4R \sin^2 \frac{\delta}{2}}{(1 - R)^2 + 4R \sin^2 \frac{\delta}{2}}$$

$$I_r = I_0 \frac{F \sin^2 \frac{\delta}{2}}{1 + F \sin^2 \frac{\delta}{2}}$$

The F is called the coefficient of Finesse,

$$F = \frac{4R}{(1 - R)^2}$$

Similarly, we can prove that the intensity of the transmitted system will be

$$I_t = I_0 \frac{1}{1 + F \sin^2 \frac{\delta}{2}}$$

Since, $\frac{I_t}{I_0} = \left(\frac{A_t}{a}\right)^2$.

From the equations of I_r and I_t it can easily be seen that, $I_r + I_t = I_0$.

The intensity distribution in the reflected part and transmitted part is complementary to each other. Thus, the two fringe systems are complimentary to each other. From Figure 13, a plot between I_r/I_0 and δ , the fringes become sharper as the values of F increases.