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# INTERFERENCE

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## 1.1 INTERFERENCE

When two (or more than two) waves of the same frequency travel almost in the same direction and have a phase difference that remains constant with time, the resultant intensity of light is not distributed uniformly in space.

*The phenomena of non-uniform re-distribution of the light intensity due to superposition of two light waves of same frequency travelling parallel to each other in same direction having constant or zero phase difference is known as **interference**.*

If the resultant intensity is zero or less than the intensities of individual waves, we have destructive interference, while if it is greater than the intensities of individual waves, we have constructive interference.

Whether the interference is constructive or destructive depends on the relative phase of the two waves. In general, if the path difference is zero or integral multiple of wavelength, the interference will be *constructive*

$$\Delta = m\lambda \quad \text{where, } m = 0, 1, 2, \dots \quad (1.1)$$

otherwise, if it is half integral multiple of wavelength, the interference would be *destructive*.

$$\Delta = \left(m + \frac{1}{2}\right)\lambda \quad \text{where, } m = 0, 1, 2, \dots \quad (1.2)$$

When two light waves are made to interfere, we get alternate dark and bright bands of a regular or irregular shape. These are called **interference fringes**.

### 1.1.1 CONDITIONS FOR INTERFERENCE

The conditions for interference of light may be divided into following subclasses:

#### **(A) Conditions for Sustainable Interference**

1. The sources must be coherent, i.e. they must be of same frequency and the phase difference between them must remain constant with time.
2. The sources must be monochromatic, i.e. they must have single frequency and must originate from the same source.
3. The sources must be in the same state of polarization.

#### **(B) Conditions for Good Contrast**

1. The amplitudes of the light waves should be equal or nearly equal.
2. The sources should be as narrow as practically possible.

#### **(C) Conditions for Good Observations**

1. The separation between two sources should be as small as possible.
2. The distance between source plane and screen should be quite large.

### 1.1.2 COHERENT SOURCES

*Two sources are said to be coherent if they emit continuous light waves of the same frequency, almost same amplitude and should either be in the same phase or have constant phase difference with respect to one another.*

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The phase relation between the waves at the time of emission rapidly and randomly changes with time, not only in different sources but even in different parts of the same source. It is thus not possible to produce interference with two such independent sources which can maintain coherency for not more than a fraction of second.

In actual practice, two virtual sources formed from a single source can be realized as coherent sources. Examples are:

1. **Young's Double Slit:** Two narrow slits  $S_1$  and  $S_2$  receive light from the same slit  $S$  produced by diffraction of light.
2. **Fresnel's Biprism:** Two virtual images of the same sources produced by reflection
3. **Lloyd's Mirror:** A real source and its virtual image produced by reflection.
4. **Newton's Ring:** By dividing the amplitudes of a portion of the wave into two parts.
5. **Michelson's Interferometer:** By reflection or refraction or both.

## 1.1.3 CLASSIFICATION OF INTERFERENCE PHENOMENON

The technique for producing two interfering beams of light from the same source may be classified under the two heads.

### (A) Division of Wavefront

This technique divides the incident wave front into two parts by utilizing the phenomena of reflection, refraction or diffraction in such a way that after covering different paths they reunite at small angle to produce interference pattern. It employs either a point source or a line source. The Young's double slit, Fresnel's biprism, Lloyd's mirror are examples of this class.

### (B) Division of Amplitude

This technique divides the amplitude of the incoming beams of light into two or more parts by utilizing the phenomena of partial reflection or refraction and give rise to two or more beams which are made to reunite to produce interference pattern. In addition to point source and line source, it may employ the broad sources too. The Newton's ring, Michelson interferometer, Fabry-Perot interferometer or etalon are example of this class.

## 1.2 FRESNEL'S BIPRISM

**(A) Construction:** A Fresnel's biprism is essentially two prisms, each of very small refracting angles placed base to base. In practice the biprism is constructed from a single plate of glass. The obtuse angle of the biprism is about  $179^\circ$  and the other angles of  $\frac{1}{2}^\circ$  are equal (Figure 1). The essential idea of biprism is to divide the incident beam into two coherent interfering beams by utilizing the phenomena of refraction hence even this single prism is termed as **biprism**.

**(B) Principle:** Monochromatic light from a narrow slit  $S$  falls on a double glass prism arranged as in Figure 2. Two virtual images  $S_1$  and  $S_2$  are formed of  $S$ , one by reflection at each half of the prism and these acts as coherent sources which are close together because of the small refracting angles ( $\frac{1}{2}^\circ$ ) of the prism. An interference pattern known as *fringes* (alternate dark and bright equally spaced vertical bands of equal thickness) is obtained in the shaded region where the two refracted beams overlap. The fringes can be observed on a screen or by using a small telescope attached with micrometer, by keeping the fringes at the cross-wire of the eyepiece.

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**(C) Theory:** An expression for the separation of two bright or dark fringes can be obtained from Figure 2. The path difference between waves reaching  $O$  from  $S_1$  and  $S_2$  is zero, i.e.  $S_1O = S_2O$ , and they therefore arrive in phase, and so there is bright fringe at  $O$ , in the center of the pattern. At  $P$ , distance  $x_m$  from  $O$ , there will be a bright fringe if the path difference at integral multiple of wave length of light, i.e., if

$$S_2P - S_1P = m\lambda$$

where  $m$  is an integer (showing the number of fringe) and  $\lambda$  is the wave length of monochromatic source. We can say that  $m^{\text{th}}$  bright fringe is formed at  $P$ . If  $d$  is the distance from the screen to the slit and  $a$ , is the separation between two virtual images of source. Hence,

$$(S_2P)^2 = d^2 + (x_m + a/2)^2 = d^2 + x_m^2 + ax_m + (a^2/4)$$

$$(S_1P)^2 = d^2 + (x_m - a/2)^2 = d^2 + x_m^2 - ax_m + (a^2/4)$$

Therefore,

$$(S_2P)^2 - (S_1P)^2 = 2ax_m$$

But

$$(S_2P)^2 - (S_1P)^2 = (S_2P + S_1P)(S_2P - S_1P)$$

In practice,  $a$  is very small compared with  $d$  and if  $P$  is near  $O$   $S_2P$  and  $S_1P$  are each just greater than  $d$ . Therefore

$$(S_2P + S_1P) \cong 2d$$

It follows that,

$$(S_2P - S_1P) = \frac{ax_m}{d}$$

For the  $m^{\text{th}}$  bright fringe at  $P$ , we have

$$m\lambda = \frac{ax_m}{d}$$

If the next bright fringe (i.e. the  $(m+1)^{\text{th}}$  bright fringe) is formed at  $Q$ , where  $OQ = x_{m+1}$ ,

$$(S_2Q - S_1Q) = (m + 1)\lambda$$

$$(m + 1)\lambda = \frac{ax_{m+1}}{d}$$

On subtraction,

$$\lambda = \frac{a}{d}(x_{m+1} - x_m)$$

If  $y$  is the distance between two adjacent bright or dark fringes, called the **fringe width**,  $y = (x_{m+1} - x_m)$ ; and so

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$$\lambda = \frac{a}{d}y$$

and therefore, fringe width  $y$  is

$$y = \lambda \frac{d}{a}$$

The above discussion is equally applicable to the dark fringes. The fringe width obtained in this way will be same as it was obtained above. The only difference will be the path difference at  $P$  and  $Q$ , which will be half integral multiple of wave length in case of dark fringes.

(D) **Measurement of Wavelength of light:** It is clear that the wave length of light can be determined only if fringe width  $y$ , separation between two virtual sources and the distance between the source slit and screen  $d$  are known.

The fringe width can be easily determined by means of a micrometer attached to the eyepiece. Adjust the vertical cross wire on a bright fringe. Take the reading and then the eye piece is moved laterally so that the vertical cross wire coincides with successive bright fringes and the corresponding readings are noted. From these reading the fringe width  $y$  can be determined.

Next, take the reading of position of the slit and the eyepiece on the optical bench. The difference between these readings gives  $d$ . The separation between two virtual sources can be determined by any of the following two methods.

(i) **Displacement Method:** The separation  $a$  can be determined by placing a convex lens between the biprism and the eyepiece. For a fixed position of the eyepiece and biprism there will be two position of the lens shown in Figure 3, where the images of  $S_1$  and  $S_2$  can be seen at the eyepiece. Let  $a_1$  be the distance between the two images when the lens is at the position  $L_1$ , i.e. at a distance  $d_1$  from the slit and  $d_2$  from the eyepiece. Hence,  $u = d_1$  and  $v = d_2$ , where  $d = d_1 + d_2$ ; therefore

$$\frac{a_1}{a} = \frac{d_2}{d_1}$$

Let  $a_2$  be the distance between the two images when the lens is at the position  $L_2$ , i.e. at a distance  $d$  from the slit and  $d_2$  from the eye piece. Hence,  $u = d_2$  and  $v = d_1$ , where  $d = d_1 + d_2$ ; therefore,

$$\frac{a_2}{a} = \frac{d_2}{d_1}$$

Then it can be easily shown that;

$$a = \sqrt{a_1 a_2}$$

Once  $a$ ,  $d$  and  $y$  is known the wavelength of light can easily be determined by using,

$$\lambda = \frac{a}{d}y$$

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- (ii) **Deviation Method:** The distance between two virtual sources can be found by using the fact that for a prism of very small refracting angle, the deviation  $\delta$  produced in a ray is given by

$$\delta = (n - 1)\alpha$$

Where  $n$  is the refraction index of the material of the prism and  $\alpha$  is the refracting angle. From Figure 4, let the distance between two virtual sources be  $a$  then

$$\tan \delta = \frac{a/2}{b}$$

$$a = 2b\delta \quad (\text{as } \delta \text{ is small, so } \delta \approx \tan \delta)$$

where  $b$  is the distance between the slit and biprism. Hence,

$$a = 2b(n - 1)\alpha$$

where  $\alpha$  is in radians.

### 1.3 INTERFERENCE BY THIN FILM

Let us consider the oblique incidence of the plane wave on the thin film. The wave reflected from the upper surface of the film interferes with the wave reflected from the lower surface of the film. Clearly the latter covers an additional optical path  $\Delta$ , which is given by (see Figure 6);

$$\Delta = n(BD + DF) - BC$$

where  $n$  is the refractive index of thin film and  $C$  is the foot of perpendicular from point  $F$  on  $BG$ . Let  $\theta$  and  $\theta'$  denote the angle of incidence and angle of refraction respectively. Then,

$$\angle JBD = \angle BDN = \angle NDF = \theta'$$

where  $N$  is the foot of perpendicular drawn from the point  $D$  on  $BF$ . Now

$$\angle BDJ = \frac{\pi}{2} - \theta' = \angle B'DJ$$

Thus,  $BD = B'D$ , and  $BJ = JB' = d$ , or  $BD + DF = B'D + DF = B'F$ . Hence

$$\Delta = nB'F - BC$$

Now,  $\angle CFB = \angle CBX = \theta$ . Thus,

$$BC = BF \sin \theta = \frac{KF}{\sin \theta'} \sin \theta = nKF$$

where  $K$  is the foot of the perpendicular drawn from  $B$  on  $B'F$ . On substitution

$$\Delta = nB'F - nKF = nB'K$$

$$\Delta = 2nd \cos \theta'$$

which is known as the **cosine law**.