Lecture Notes Dated: Aug 31-Sep 01, 2012

The Carnot Engine

Heat Engine and its Efficiency

Carnot observed that the function of the heat engine is to extract a certain amount of heat from the heat reservoir, convert a part of it to work and transfer the rest to the heat sink. The heat engine has to operate continuously in cycle. He also showed how these operations should be carried out so that the efficiency may be maximum.

The efficiency of an engine is defined as the ratio of the net work done to the heat absorbed during one complete cycle. It is denoted by the symbol η.

$$\eta = \frac{useful\ work\ done}{heat\ absorbed}$$

After one complete cycle, there will be no change in its internal energy, i.e., $dU = 0$. Thus

$$\delta W = \delta Q \quad \text{i.e.,} \quad W = Q_1 - Q_2$$

where Q_1 is the heat absorbed, Q_2 is the heat rejected and W is the work done during one cycle. Therefore, we have

$$\eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1}$$

or,
$$\eta = 1 - \frac{Q_2}{Q_1} \tag{20}$$

This equation shows that η will always be less than unity. It remains independent of working substance, filled in the cylinder of engine.

Figure 4 *(a) Carnot engine,* *(b) Carnot cycle.*

Carnot Cycle and Carnot Engine

An ideal engine operating in a reversibile cycle, in which there are no dissipative losses to waste energy, known as the ***Carnot cycle***. A Carnot cycle is a set of processes that can be performed by any thermodynamic system.

An engine operating in a Carnot cycle is called a ***Carnot engine***. A Carnot engine, shown in fig. 4, operates between two reservoirs in a particularly simple way. The Carnot engine is a reversibile engine. The function of the Carnot engine is to extract a certain amount of heat from the heat reservoir, convert a part of it to work and transfer the rest to the heat sink. It is an ideal theoretical engine free from all the imperfectness of actual engine, and hence never realized in actual practice.

Carnot Cycle and its Efficiency

The Carnot cycle consists of four set of processes, which operates through the following ideal arrangement. The F is a heat reservoir at temperature T_1 (fig. 5), G is a heat sink at T_2, H which is simply a nonconducting cap, S is the cylinder of the engine containing a perfect gas as the working substance and fitted with a nonconducting piston. The walls of the cylinder are adiabatic, but the bottom is perfectly conducting. The behavior of the working substance is shown by the *p-V* diagram showing the pressure and the volume of the gas at any instant. Let the following steps be performed.

Figure 5 *The ideal Carnot engine with indicator diagram.*

(1) Reversible Isothermal Expansion. Let the initial temperature of the gas within cylinder be T_1 and let it be placed in contact with heat reservoir, and the piston moved forward slowly. As the piston moves, the temperature tends to fall, and heat will pass from heat reservoir to cylinder. The operation is performed very slowly, so that the temperature of the gas is always T_1. The representative point on the indicator diagram moves from A to B, along an isothermal curve. The heat Q_1 is extracted in this process is equal to the work done by the gas(For an isothermal process $W_1 = Q_1$, because $dU = 0$, and $pV = RT_1$) in this reversible isothermal expansion, and is given by

$$W_1 = Q_1 = \int_A^B p\,dV = RT_1 \int_A^B \frac{dV}{V}$$

$$W_1 = Q_1 = RT_1 \ln \frac{V_B}{V_A} = area\ \boldsymbol{AabB} \qquad (21)$$

where V_A and V_B are volume of the gas in states A and B.

(2) Reversible Adiabatic Expansion. The heat reservoir is then removed and a nonconducting cap is applied to the cylinder, and the piston allowed to move forward by inertia. Then the gas will describe the adiabatic BC and will fall in temperature. We stop at C when the temperature has fallen to T_2. The work done by the gas in this reversible adiabatic expansion is given by

$$W_2 = \int_B^C p\,dV = K \int_B^C \frac{dV}{V^\gamma}$$

$$W_2 = \frac{K}{1-\gamma}\left[\frac{1}{V_C^{\gamma-1}} - \frac{1}{V_B^{\gamma-1}}\right] = \frac{1}{1-\gamma}(p_C V_C - p_B V_B)$$

$$W_2 = \frac{R(T_1 - T_2)}{\gamma - 1} = area\ \boldsymbol{BbcC} \qquad (22)$$

where $pV^{\gamma}=K=p_C V_C^{\gamma}=p_B V_B^{\gamma}$ and V_B and V_C are volume of the gas in states B and C.

Since the pressure is now very low, the gas can not expand further, hence in order to bring back its capacity for doing work it must be brought back to its original condition. To effect this we compress the gas in two stages;

(3) Reversible Isothermal Compression. The cylinder is now placed in contact with the heat sink at temperature T_2 and compress the gas isothermally along the path CD on indicator diagram. The heat Q_2 which is developed due to compression will now pass to the sink. This heat is equal to the work done on the gas and is equal to

$$W_3=-Q_2=\int_C^D pdV=RT_2\int_C^D \frac{dV}{V}$$

$$W_3=-Q_2=RT_2\ln\frac{V_D}{V_C}=area\ \boldsymbol{CcdD} \qquad (23)$$

where V_C and V_D are volume of the gas in states C and D. Since $V_C > V_D$, equation signifies that W_3 is negative, i.e., work is done on the gas.

(4) Reversible Adiabatic Compression. The cylinder is now placed in contact with the nonconducting cap and the gas is compressed adiabatically along the path DA on indicator diagram, unless the gas again restores its initial temperature T_1.

$$W_4=\int_D^A pdV=K\int_D^A \frac{dV}{V^{\gamma}}$$

$$W_4=\frac{K}{1-\gamma}\left[\frac{1}{V_A^{\gamma-1}}-\frac{1}{V_D^{\gamma-1}}\right]=\frac{1}{1-\gamma}(p_A V_A-p_D V_D)$$

$$W_4=\frac{R(T_2-T_1)}{\gamma-1}=-W_2=area\ \boldsymbol{DdaA} \qquad (24)$$

where $pV^{\gamma}=K=p_A V_A^{\gamma}=p_D V_D^{\gamma}$ and V_A and V_D are volume of the gas in states A and D.

The net effect is that we have extracted heat Q_1 from heat reservoir at T_1 and put back Q_2 to heat sink at T_2. The net work done by the engine in one complete cycle,

$$W=W_1+W_2+W_3+W_4=area\ \boldsymbol{ABCD} \qquad (25)$$

It is seen that $W_4 = -W_2$, so we have

$$W=W_1+W_3=Q_1-Q_2=RT_1\ln\left(\frac{V_B}{V_A}\right)+RT_2\ln\left(\frac{V_D}{V_C}\right) \qquad (26)$$

Since B and C lie on the same adiabatic, we have by the eq. (5.19a)

$$T_1 V_B^{\gamma-1}=T_2 V_C^{\gamma-1} \qquad \text{or,} \quad \frac{V_C}{V_B}=\left(\frac{T_1}{T_2}\right)^{1/(\gamma-1)} \qquad (27)$$

Similarly, D and A lie on the same adiabatic, we have

$$T_2 V_D^{\gamma-1}=T_1 V_A^{\gamma-1} \qquad \text{or,} \quad \frac{V_D}{V_A}=\left(\frac{T_1}{T_2}\right)^{1/(\gamma-1)} \qquad (28)$$

Hence, $$\frac{V_C}{V_B}=\frac{V_D}{V_A} \qquad \text{or,} \quad \frac{V_D}{V_C}=\frac{V_A}{V_B} \qquad (29)$$

Therefore by eq. (26), we have

$$W = Q_1 - Q_2 = RT_1 \ln\left(\frac{V_B}{V_A}\right) + RT_2 \ln\left(\frac{V_A}{V_B}\right)$$

$$W = Q_1 - Q_2 = R(T_1 - T_2)\ln\left(\frac{V_B}{V_A}\right) \quad (30)$$

$$Q_1 = RT_1 \ln\left(\frac{V_B}{V_A}\right) \quad \text{and} \quad Q_2 = RT_2 \ln\left(\frac{V_B}{V_A}\right) \quad (31)$$

Hence,
$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2} = \frac{W}{T_1 - T_2} = R\ln\left(\frac{V_B}{V_A}\right) \quad (32)$$

or,
$$W = Q_1\left(\frac{T_1 - T_2}{T_1}\right)$$

Thus, the efficiency of the Carnot engine is

$$\eta = \frac{W}{Q_1} = \frac{T_1 - T_2}{T_1}$$

or,
$$\eta = 1 - \frac{T_2}{T_1} \quad (33)$$

After analysing the mode of operation of the Carnot cycle, we can draw the following conclusions about Carnot engine:

1. Carnot cycle is just a set of processes consisting of two isothermal and two adiabatic processes, reversible at each stage.
2. No engine can be more efficient than the Carnot engine.
3. The nature of the working substance is immaterial. In fact it is temperature difference that actually matters. By eq. (5.32), we further find that

$$\frac{T_1}{T_2} = \frac{Q_1}{Q_2} \quad (34)$$

i.e., for an ideal gas, the ratio Q_1/Q_2 also depends on temperature T_1 and T_2.