Lecture Notes Dated: Nov 10 & 17, 2012

The Principle of Regenerative Cooling

The Joule-Thomson cooling observed for mos gases is very small. Thus for air at a temperature of 20ºC when the pressure on the two sides are 50 atmospheres and 1 atmosphere respectively. It was found that the temperature falls by 11.7ºC. Even using a pressure of 210 atmospheres, the fall in temperature is only 42ºC. But the effect is intensified by undergoing a process known as *regenerative process*.

A portion of the gas which first suffers Joule-Thomson expansion and becomes cooled is employed to cool other portions of the incoming gas before the later reaches the nozzle. In this way the cooling effect can be multiplied by many times. This cumulative process used to cool a gas continuously is called the *regenerative cooling*.

In practice, this is achieved by using concentric tubes. Two or more concentric tubes are arranged in the form of spirals, the inner one carrying the high pressure inflowing gas while the outer one the low pressure outflowing gas. In the regenerative method, as lower the initial temperature the greater is the Joule-Thomson cooling.

Figure 3 *Illustration of the principle of regenerative cooling.*

The regenerative principle is illustrated in figure 3, where the high pressure gas from the compressor enters the spirals contained in the water-cooled jacket A. The gas then enters into the regenerator coils at E and by expansion at the valve C becomes cooled by a small amount. This returns by the outer tube abstracting heat from the high pressure gas and reaches F almost at the same temperature as the incoming gas at E. As time passes, the gas approaching compressor is again compressed, cooled by A and re-enters at E. After some time, the gas approaching C becomes cooled more and more till the Joule-Thomson cooling at C is sufficient to liquefy it, viz., its temperature reaches the value at which the gas would liquefy under the pressure prevailing at

F. A portion of the escaping gas then condenses inside the Dewar flask D.

Figure 4 *Temperature distribution in regenerative cooling.*

At this stage the temperature throughout the apparatus become steady and may be represented by the curve shown in figure 4. The part LM represents continuous decrease of temperature of the gas as we approach the nozzle through the inner tube while MN represents the Joule-Thomson cooling. The NL represents the temperature of the low pressure gas which is less than that of the adjacent high pressure gas at every point of the coil. Thus the cooler low pressure gas abstracts heat from the incoming stream. In the ideal gas, the temperature of the low pressure gas at F should be the same as that of the high pressure gas at the entrance E.

Thus, if the J-T cooling can be used regeneratively, it ultimately leads to production of temperature low enough for liquefaction of air.

Refrigeration Cycle

Carnot Refrigerator

Since a Carnot cycle consists of reversible processes, it may be performed in either direction. When it is performed in a direction opposite to that shown in the examples of Carnot cycle, then this cycle is known as a ***refrigeration cycle*** and the system is called a ***Carnot refrigerator*** which is represented symbolically in figure 5.

Figure 5 *The Carnot Refrigerator*

The important feature of a Carnot refrigeration cycle, which separate it from any other reversible engine cycle, is that the heat given to heat reservoir Q_1, heat extracted from heat sink Q_2 and work done on the working substance W are numerically equal to those quantities when the cycle is performed in the opposite direction, i.e., in Carnot engine.

Practical Refrigerator

Thus, the cycle working in the reverse direction will act as an ideal *refrigerator* in which heat is taken from the heat sink, some work is performed on it and finally heat is transferred to the heat reservoir. Thus, we can consider refrigerator as a machine which is capable of transferring heat from the low temperature to the high temperature with the aid of external work.

The working substance in the refrigerator is known as *refrigerant*. Any liquid can be used as a refrigerant—water, ammonia, sulphur dioxide, carbon dioxide, freon, etc. but in practice the choice is very limited.

Figure 6 *The arrangement of practical refrigerator.*

The arrangement is shown in figure 6 A practical refrigerator consists four principle parts;

1. A compressor G, usually electrically driven, which is fed by refrigerant boiling at reduced pressure from an evaporator E through an intake valve.
2. The gas is compressed adiabatically and delivered to the condenser Q, which is generally a coil of copper embedded in a vessel through which some coolant (cold water or brine) is passing. The heat of compression is carried off by the coolant, and the vapour becomes liquefied.
3. The liquid refrigerant now passes to the reservoir R, from which through an throttling valve V, it is deliver to the evaporator E, and another refrigeration cycle starts.
4. The evaporator coils E are embedded in the refrigerator system.

Working of a Practical Refrigerator

The refrigerant in the form of liquid is stored in the reservoir R at a temperature equal to that of the condenser Q whose temperature is maintained by the coolant working as heat reservoir. The liquid from reservoir R passes through throttling valve suffering J-T expansion and cools to a lower temperature. Then it reaches to the evaporator E absorbing required latent heat from the object to be cooled (working as sink) and converted into vapour at reduced pressure.

These vapours is sucked by the compressor G in its intake stroke through an intake valve. The vapour is then compressed adiabatically and delivered to the condenser Q through an outtake valve, where the coolant carried off the heat of compression and the vapour again liquefied, and the next cycle starts.

In this way the refrigerant takes away the heat from the object and reject it to the coolant with the aid of work performed by the electrically fed compressor.

Efficiency of a Refrigerator

The ***efficiency*** or ***coefficient of performance*** for a refrigerator is defined as

$$\omega=\frac{heat\,taken\,from the\,sink}{work\,input}$$

$$\omega=\frac{Q_2}{W}=\frac{Q_2}{Q_1-Q_2}=\frac{T_2}{T_1-T_2} \tag{11}$$

The w can be much greater than unity.

If η is the efficiency of a Carnot engine operating between the same temperatures, we have

$$\eta=1-\frac{T_2}{T_1}=\frac{T_1-T_2}{T_1}$$

$$\frac{1}{\eta}=\frac{T_1}{T_1-T_2}$$

$$\frac{1}{\eta}-1=\frac{T_1}{T_1-T_2}-1$$

$$\frac{1-\eta}{\eta}=\frac{T_1-T_1+T_2}{T_1-T_2}=\frac{T_2}{T_1-T_2}=\omega$$

Therefore, the coefficient of performance in terms of efficiency of Carnot engine is given by

$$\omega=\frac{1-\eta}{\eta} \tag{12}$$

No refrigerator can have a higher coefficient of performance than the Carnot refrigerator, if both are working between same reservoirs. Any practical refrigerator can have ω between 2 to 6.

Cooling due to Adiabatic Demagnetization

Adiabatic demagnetization is a very powerful method of producing temperature lower than the helium limit. In 1926 it was suggested independently by Debye and by Giauque. From theoretical considerations they showed that a paramagnetic salt may be made to do work to accomplish temperature below 1K.

Principle

The principle of the method is illustrated in figure 7, where we have plotted entropy of iron-ammonium alum as a function of temperature for zero field and 15 kG field.

Figure 7 *T-S curve for a paramagnetic salt: Principle of adiabatic demagnetisation.*

We observe that the entropy of the specimen is more when there is no magnetic field. Due to thermal energy, the spins of a paramagnetic substance are randomly oriented. But when a magnetic field is applied, these spins are partially aligned along the field and entropy of the specimen decreases, because the entropy of the ordered system is smaller.

Like adiabatic expansion, adiabatic demagnetization is a two stage process. In the first stage the paramagnetic salt, in contact with a heat bath at about 1K, is adiabatically magnetised by switching on a magnetic field. During this stage, no heat is allowed to leave the system the internal energy of the specimen increases and its temperature rises. The heat produced is transferred from the paramagnetic specimen to the heat bath. The specimen is now thermally isolated and the applied magnetic field is switched off adiabatically and reversibly. As no heat is allowed to enter the system the internal energy of the specimen decreases and its temperature falls.

Production and Measurement of Low Temperatures

After theoretical explanation, Giauque demonstrated the cooling by adiabatic demagnetization experimentally. Figure 8 illustrates the experimental procedure. This is as follows:

1. The paramagnetic salt, say *Gadolinium sulphate* $Gd_2(SO_4)_3.8H_2O$ is taken in the form of a powder and is first pressed into the form of a cylinder. It is then suspended on silk or nylon fibres inside a cylindrical chamber which is surrounded by a liquid helium bath and the later vigorously pumped so that a temperature a temperature of about 1K is obtained. During the pumping the salt is cooled down to the bath temperature by the exchange gas, introduced previously inside the cylindrical chamber. The bath itself is within the poles of an electromagnet.
2. The magnetic field is now switched on, and the heat of magnetisation is conducted away by gaseous helium in the chamber to the helium bath outside.
3. The chamber is now evacuated by a powerful pump to the highest vacuum, and the specimen is thus thermally isolated.
4. The field is now entirely switched off, or reduced. The temperature of the specimen now falls. This fall in ***temperature is measured*** by determining the change in the paramagnetic susceptibility of the specimen.
5. To measure this quantity, the sample is surrounded by a close fitting coaxial solenoid and the self-inductance of the coil is measured with the Anderson Bridge. The value of the inductance gives the susceptibility.
6. From ***Curie law*** we know that for many paramagnetic substance, the magnetic susceptibility is given by, $\chi = C/T$ where C is a constant, called *Curie constant*, and T is the absolute temperature.
7. The Gadolinium is one of the best studied substance whose salts were studied by *K. Onnes* up to 1.3K, and were found to obey the law $\chi = C/T$. But generally, a modified law $\chi = C/(T - T_C)$ due to *Weiss* is found to give better accord with experiments, and in the case of Gd, up to 1.3K, where T_C is *Curie temperature*.

Figure 8 *Apparatus for producing cooling by adiabatic demagnetisation.*