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Blackbody

The ability of a body to radiate is closely related to its ability to absorb radiation. This is to be expected, since a body at a constant temperature is in thermal equilibrium with its surroundings and must absorb energy from them at the same rate as it emits energy. It is convenient to consider as an ideal body one that absorbs all radiation incident upon it, regardless of frequency, and emit a radiation of universal character, but does not depend on the composition of the object. Such a body is called ***blackbody***.

A perfectly blackbody is one which absorb all the radiations of whatever frequency, incident on it. It neither reflects nor transmits any of the incident radiation and therefore, appears black whatever be the colour of radiation. Since all blackbodies behave identically, we can now disregard the precise nature of whatever is radiating.

In practice, no substance possesses strictly the properties of a blackbody. Lamp black and the platinum black are the nearest approach to a blackbody. However, the bodies showing close approximation to a perfectly blackbody can be constructed. The blackbody due to Fery and the blackbody due to Wien are examples of a perfectly blackbody for experimental purpose.

Radiation in a Hollow Enclosure

In general, the nature of radiation and the amount of radiation per unit area per unit time, *i.e.*, the quality and the quantity of radiation depends on the temperature as well as on the nature of the surface of radiating body.

If however, we consider a nonconducting enclosure maintained at a constant temperature, i.e., nonconducting isothermal enclosure ; the quality and the quantity of radiation inside such an enclosure depend only on its temperature and are entirely independent of both the nature of the walls and the nature of the substances which may be inside it. Thus, we can say that, *the energy density of the radiation in a uniform temperature enclosure depends on its temperature and is totally independent of the nature of the walls.*

Now, let us consider that the bodies A, B and C are placed inside an isothermal enclosure. Experience shows that each body will attain the temperature of the enclosure irrespective of its nature or original temperature. As the bodies are at the same temperature as that of the enclosure, each body must radiate energy at the same rate and exactly of same kind as its absorbs, each body must radiate energy at the same rate and exactly of same kind as it absorbs. In other words, we can say that, *the bodies contained inside the enclosure have no effect on the quantity or quality of the radiant energy filled in the enclosure.*

Pure Temperature Dependence

Let a blackbody be placed in an isothermal enclosure. The body will emit the full radiation of the enclosure after it is in thermal equilibrium with the enclosure. These radiations are independent of the nature of the substance. Clearly the radiation from an isothermal enclosure is identical with that from a blackbody at the same temperature. Therefore, the heat radiations in an isothermal enclosure are termed as ***blackbody radiation***, and this radiation *purely depends on its temperature*. This characteristics of blackbody is termed as ***pure temperature dependence***.

Blackbodies in Practice

We have seen that if an enclosure be maintained at a constant temperature it becomes filled with radiation characteristic of a perfectly blackbody. A metallic cavity with a small hole is an example of a blackbody. Radiation entering the hole is eventually absorbed after successive reflections at the inner walls of the cavity. Some of the radiation re-emitted by the atoms in the walls of the cavity leaks out through the hole. This radiation (***blackbody radiation***) has the same character as that of any other perfect blackbody.

A particular type of ***blackbody due to Fery*** (Figure 1) consists of a hollow copper sphere blackened inside with a very fine hole in the surface. When the radiations enter through the hole, they suffer a number of reflections at the inner walls of the sphere until it is completely absorbed. To avoid direct reflection from the surface opposite to the hole, a conical projection is made in front of the hole. Thus the hole acts as a blackbody absorber. When this sphere is placed in a bath such as water bath at a fixed temperature, the heat radiations comes out the hole. Thus only the hole not the walls of the sphere, acts as a black body radiator.

Figure 1 *Blackbody due to Fery.*

Figure 2 *Blackbody due to Wien.*

Another type of ***blackbody due to Wein*** consists of a hollow cylindrical metallic chamber C (Figure 2) blackened inside and made of brass or platinum, depending upon the temperature at which it would be used. The cylinder is heated by an electric current passing through this platinum foil as indicated by thick dashes. The radiation then passes through a number of limiting diaphragms and emerges out of the hole O. The cylinder is surrounded by concentric porcelain tubes. The temperature of the blackbody is given by the thermo-element T. This is the type of blackbody now widely and commonly used for experimental purposes.

Spectral Distribution of Blackbody Radiation

The Lummer and Pringsheim in 1899 studied experimentally the distribution of energy among radiation of different frequency in the radiation spectrum of a blackbody.

Experimental Arrangement to obtain Spectral Distribution

A carbon tube heated electrically serves as the perfect blackbody S. A narrow beam of radiation from the blackbody is converted into a parallel beam by the use of a concave mirror M_1 as shown in figure 3. The parallel beam of radiation, then falls on *flourspar prism* P, which disperses the heat radiation into different frequencies in a similar manner as the glass prism does in the case of white light. The different frequencies in dispersed heat radiation are brought to focus at different points with the help of another concave mirror M_2. By slowly rotating the prism the different parts of the radiation spectrum in the form of a small band of frequencies can be made to fall on a *linear bolometer* B (a device to detect the thermal radiation), which measures the energy of the radiation in terms of the deflection in galvanometer G connected in the bolometer circuit. The spectral energy associated with the different frequencies is found to be different at a particular constant temperature.

Figure 3 *Arrangement to study the blackbody radiation spectral distribution.*

The Spectrum of Blackbody Radiation

The spectrum of blackbody radiation is shown in figure 4 for two temperatures, which is a plot of spectral energy density at each frequency, versus the frequency of the radiation. The following conclusions can be drawn from these curves:

1. At each constant temperature, the blackbody emits continuous heat radiation spectrum.
2. The energy associated with the radiation of a particular frequency increases with increase in temperature of the blackbody.
3. The energy distribution is not uniform, *i.e.*, at a given temperature spectral energy density initially increases with frequency and after attaining a maxima, it decreases.
4. The frequency of maximum emission shifts towards greater frequency as the temperature of blackbody increases, i.e., ν_{max} is proportional to absolute temperature,i.e., $\nu_{max} \propto T$. This is in accordance to the **Wien's displacement law**.
5. The area enclosed between any curve and *x*-axis, which is total energy emitted per unit surface area per unit time at a temperature by a perfect blackbody is found to be proportional to the forth power of the absolute temperature, i.e., $E \propto T^4$. This is the statement of **Stefan-Boltzmann law** of radiation.

Figure 4 *spectrum of blackbody radiation.*

Explanation of Blackbody Spectrum

Why does the blackbody spectrum have the shape shown in figure 4? Several efforts have been made to explain the results obtained in the spectrum of blackbody radiation, which is an outcome of convincing experiments. We shall discuss few important of them.

Classical Explanation: Rayleigh – Jeans Law

To explain this spectrum, the classical calculation by Rayleigh and Jeans begins by considering a blackbody as a radiation filled cavity at the temperature *T*. Because the cavity walls are assumed to be perfect reflectors, the radiation must consist of standing electromagnetic waves, as in figure 5. In order for a node to occur at each wall, the path length from wall to wall, in any direction, must be an integral number of half wavelengths.

Figure 5

If the cavity is a cube of length *L* on each side, this condition means that for standing waves in the *x, y* and *z* directions respectively, the possible wavelengths are such that

$$n_x = \frac{2L}{\lambda} = 1, 2, 3, \ldots \Rightarrow \text{ number of half wavelengths in } x \text{ direction}$$
$$n_y = \frac{2L}{\lambda} = 1, 2, 3, \ldots \Rightarrow \text{ number of half wavelengths in } y \text{ direction} \qquad (1)$$
$$n_z = \frac{2L}{\lambda} = 1, 2, 3, \ldots \Rightarrow \text{ number of half wavelengths in } z \text{ direction}$$

For a standing wave in any arbitrary direction, it must be true that

$$n_x^2 + n_y^2 + n_z^2 = \left(\frac{2L}{\lambda}\right)^2 \quad \text{where} \quad \begin{matrix} n_x = 1, 2, 3, \ldots \\ n_y = 1, 2, 3, \ldots \\ n_z = 1, 2, 3, \ldots \end{matrix} \qquad (2)$$

in order that the wave end in a node at its ends.

To counts the number of standing waves $g(\lambda)d\lambda$ within the cavity whose wavelengths lie between λ and $\lambda + d\lambda$, what we have to do is count the number of permissible sets of n_x, n_y, n_z values that yield wavelengths in this interval.

Let us imagine a *n*-space whose coordinate axes are n_x, n_y, and n_z. Each point in the *n*-space corresponds to a permissible set of n_x, n_y, n_z values and thus corresponds to a standing wave also. If **n** is vector from origin to a particular point n_x, n_y, n_z, its magnitude is

$$n=\sqrt{n_x^2+n_y^2+n_z^2} \qquad (3)$$

The total number of wavelengths between λ and λ + *d*λ is the same as the number of points in *n*-space whose distances from the origin lie between *n* and *n* + *dn*. The volume of a spherical shell of radius *n* and thickness *dn* is ($4\pi n^2 dn$), but we are only consider the octant (⅛th)of this shell that includes positive values of n_x, n_y and n_z. Also, for each standing wave counted in this way, there are two perpendicular directions of vibration. Hence, the number of independent standing waves in the cavity is

$$g(n)dn=(2)(1/8)(4\pi n^2 dn)=\pi n^2 dn \qquad (4)$$

Now we consider the number of independent standing wave in the cavity as a function of their frequency *v* instead of as a function of *n*. From Eqs. (2) and (3) we have

$$n=\frac{2L}{\lambda}=\frac{2L\nu}{c} \qquad \text{and} \qquad dn=\frac{2L}{c}d\nu$$

and so, the number of standing waves in the cavity whose frequency lies between *v* and *v* + d*v* , is

$$g(\nu)d\nu=\pi\left(\frac{2L\nu}{c}\right)^2\frac{2L}{c}d\nu=\frac{8\pi L^3}{c^3}\nu^2 d\nu \qquad (5)$$

As the cavity is a cube *L* long each side, the volume is therefore L^3. Then the number of independent standing waves *G*(*v*)*dv* in the frequency interval between *v* and *v* + d*v* per unit volume in the cavity turned out to be

$$G(\nu)d\nu=\frac{1}{L^3}g(\nu)d\nu=\frac{8\pi\nu^2 d\nu}{c^3} \qquad (6)$$

The formula is independent of the shape of the cavity. The higher the frequency *v*, the shorter the wavelength and the greater the number of possible standing waves.

The next step to find the average energy per standing wave. According to the classical theorem of **equipartition of energy** the average energy per degree of freedom of an entity that is part of the system of such entities in thermal equilibrium at the temperature *T* is ½*kT*. Each standing wave in a radiation filled cavity corresponds to two degrees of freedom, for a total of 2(½)*kT*, because each wave originates in an oscillator in the cavity wall. Such an oscillator has two degree of freedom, one that represents its kinetic energy and one that represents its potential energy. Thus, classical average energy per standing wave

$$\bar{\epsilon}=kT \qquad (7)$$

The energy *u*(*v*)*dv* per unit volume in the cavity in frequency interval between *v* to *v* + *dv*, i.e., spectral energy density of blackbody is

$$u(\nu)d\nu=\bar{\epsilon}G(\nu)d\nu=\frac{8\pi kT}{c^3}\nu^2 d\nu \qquad (8)$$

The result known as the ***Rayleigh-Jeans Law***, for spectrum of blackbody radiation. This law agrees with experimental result only with lower frequencies (figure 6), but it fails miserably at higher frequencies (specially in ultraviolet region). The failure of Rayleigh-Jeans law in higher frequencies region is known as ***ultraviolet catastrophe***.